PhD Thesis Defence

KADIR HAS UNIVERSITY

SCALE AND COMPLEXITY:

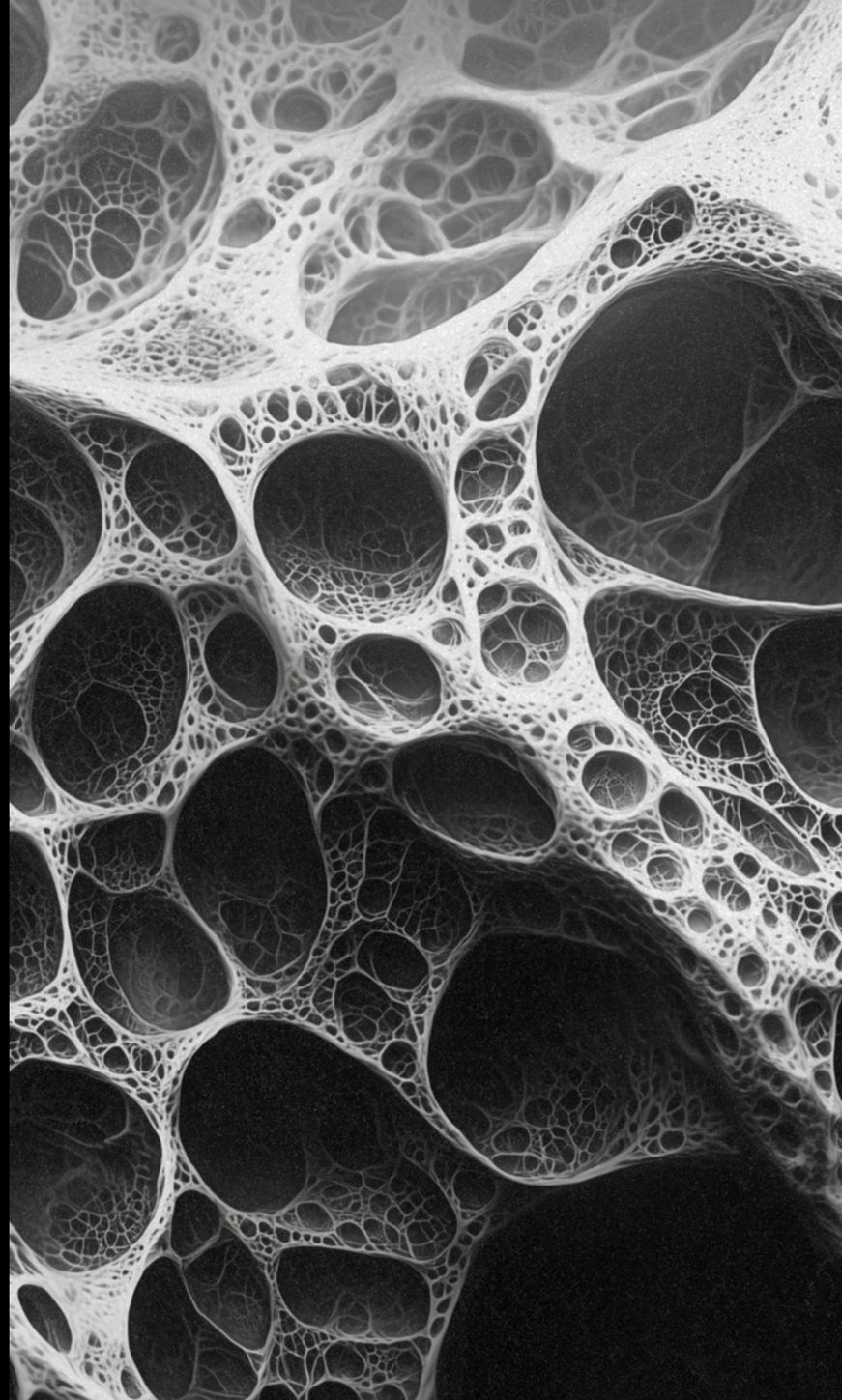
Renormalization-Group Theory of Phase Transitions, Spin-Glass Chaos, and Emergent Order in Complex Spin Systems

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Computational Sciences and Engineering

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Motivation

EMERGENCE

how do simple microscopic rules generate complex macroscopic behavior?

UNIVERSALITY

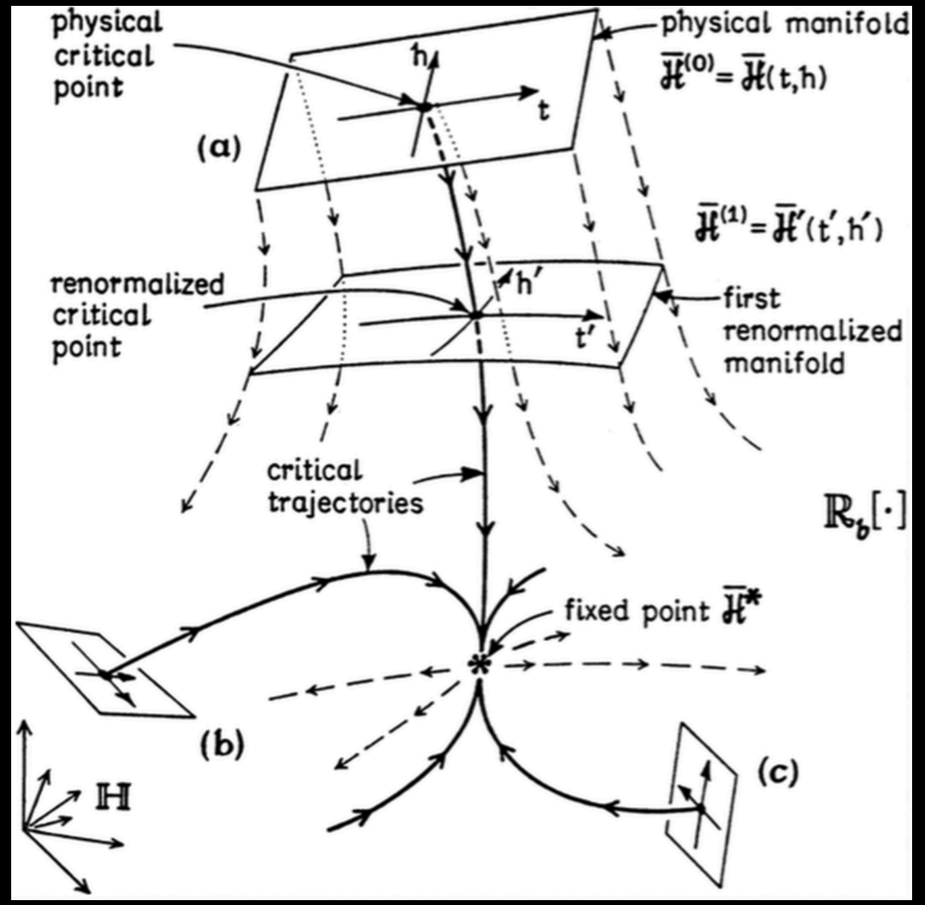
why do microscopically different systems exhibit identical macroscopic properties?

COMPLEXITY

what determines whether a system is simpler or more complex?

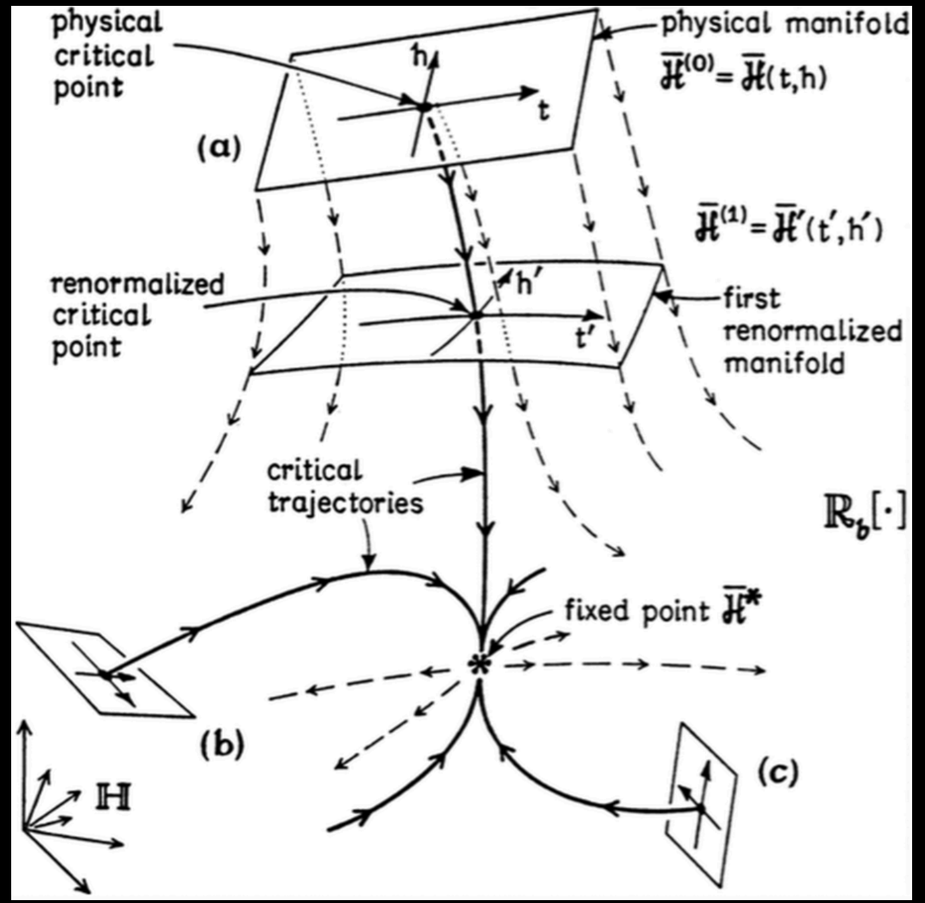
scale transformation — how descriptions change as degrees of freedom are systematically eliminated via the Renormalization Group

“complexity” is diagnosed
by the geometry of renormalization-group flows
generated by a Hamiltonian under a specified
coarse-graining scheme



RG FLOW TOPOLOGY → PHASE STRUCTURE

- attractors: fixed points, fixed lines, chaos
- phase boundaries: separatrices
- critical exponents: eigenvalues at fixed points
- phase identifications: asymptotic RG flows



Generators & Modulators of Complexity

FRUSTRATION

competing interactions → chaotic RG flows, spin-glass phases

Chs. 3-6

SYMMETRY COMPETITION

incompatible ordering → phase proliferation, algebraic order

Chs. 7-8

SCALE INVARIANCE

continuous symmetry in d=2 → fixed lines, varying exponents

Ch. 8

INTERACTION RANGE

power-law interactions tune effective dimensionality

Chs. 9-10

EXTERNAL FIELDS

break symmetry, reveal latent structure

Ch. 5

SPATIAL TOPOLOGY

fractal surfaces → independent surface chaos

Ch. 6

Outline

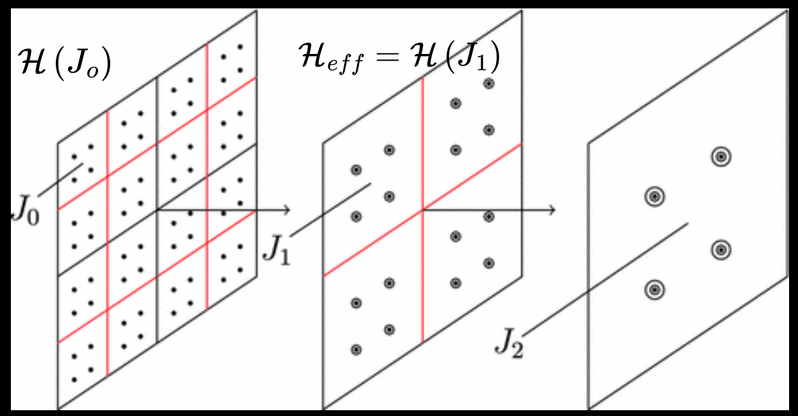


- ~~motivation & introduction~~
- methodology
- thermodynamic foundations
- frustration
  - spin-glass chaos
  - emergent order
  - phase proliferation under fields
  - surface chaos
- symmetry competition
  - merged discrete-spin systems
  - merged continuous-spin systems
- long-range interactions
- extension to non-equilibrium
- concluding remarks

Method

POSITION-SPACE RENORMALIZATION GROUP

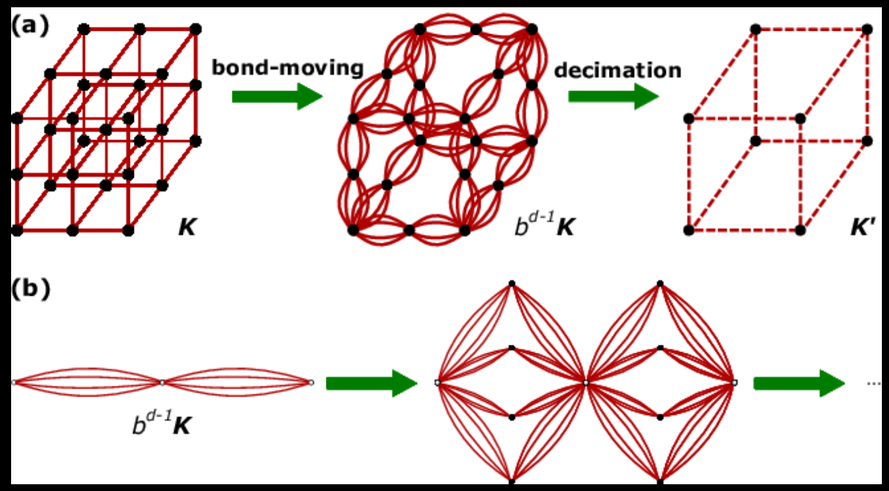
$$\frac{dJ_\alpha}{dl} = \beta_\alpha(\{J\})$$



$$Z = \sum_{\{s\}} e^{-\beta\mathcal{H}(\{s\})} = \sum_{\{s'\}} e^{-\beta\mathcal{H}'(\{s'\})} = Z'$$

MIGDAL-KADANOFF APPROXIMATION

- bond-moving – redistribute interactions
- decimation – sum over internal spins
- exact on hierarchical lattices



Implementation

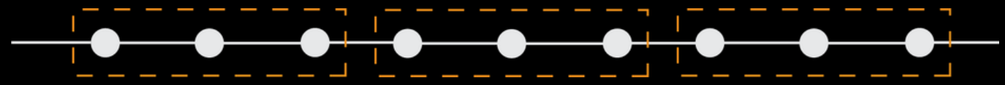
TRANSFER MATRIX REPRESENTATION

$$Z = \sum_{\{s\}} \prod_{\langle ij \rangle} e^{-\beta \mathcal{H}_{ij}(s_i, s_j)} \equiv \sum_{\{s\}} \prod_{\langle ij \rangle} T_{ij}(s_i, s_j)$$

$$T'_{ij}(s_i, s_j) = \sum_{s_k} T_{ik}(s_i, s_k)\, T_{kj}(s_k, s_j)$$

$$T = \begin{bmatrix} e^{-\beta \mathcal{H}_{11}(s_1, s_1)} & e^{-\beta \mathcal{H}_{12}(s_1, s_2)} \\ e^{-\beta \mathcal{H}_{21}(s_2, s_1)} & e^{-\beta \mathcal{H}_{22}(s_2, s_2)} \end{bmatrix}$$

NIEMEIJER-VAN LEEUWEN CELL-SPIN METHOD



$$Z = \sum_{\{s\}} e^{-\beta \mathcal{H}(\{s\})} = \sum_{\{s'\}} \sum_{\{\sigma\}} e^{-\beta \mathcal{H}(\{s'\}, \{\sigma\})} = \sum_{\{s'\}} e^{-\beta \mathcal{H}'(\{s'\})}$$

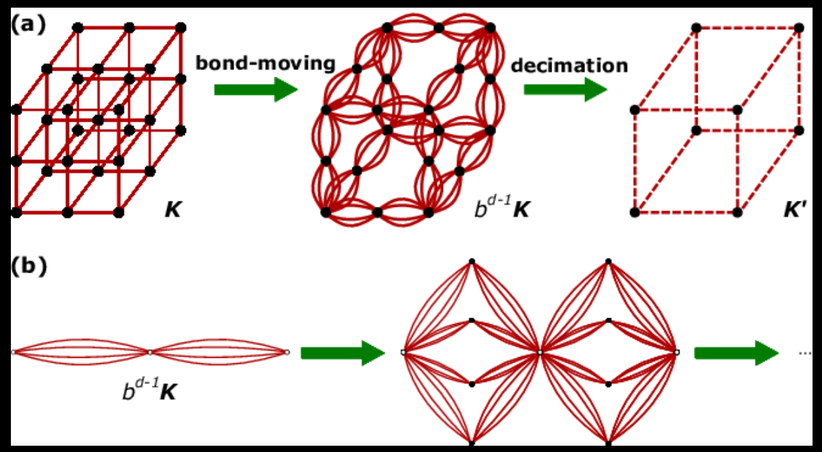
Total Thermodynamics via RG

q-STATE POTTS MODEL

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} J\delta(s_i, s_j)$$

q-STATE CLOCK MODEL

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} J\cos(\vec{s}_i \cdot \vec{s}_j)$$



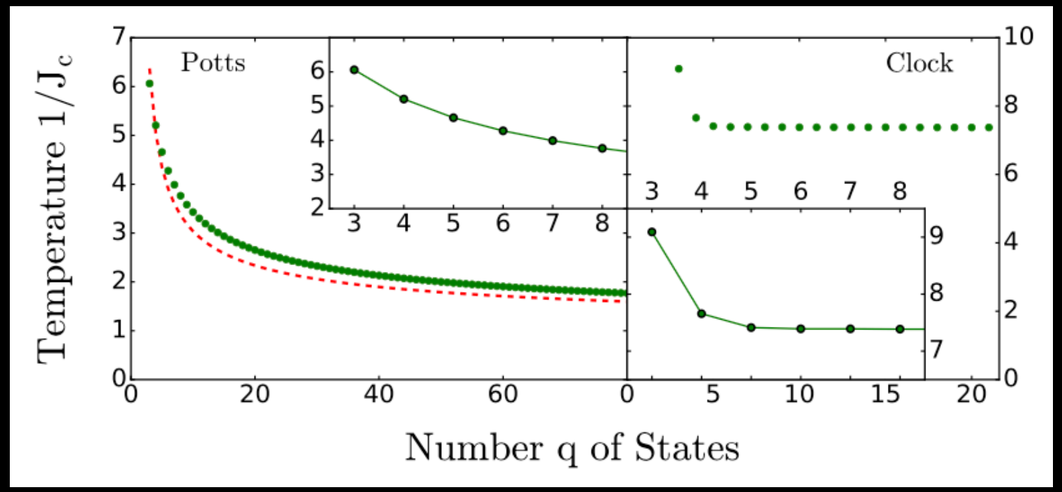
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$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} J\cos(\vec{s}_i \cdot \vec{s}_j)$$



Total Thermodynamics via RG

BOND-STATE DENSITIES

$$U(n_i, n_j) = \langle \delta(s_i, n_i)\delta(s_j, n_j) \rangle$$

$$U(n_i, n_j) = \frac{1}{N}\frac{\partial \ln Z}{\partial E(n_i, n_j)}$$

$$E(n_i, n_j) = J\delta(n_i, n_j), \quad J\cos(2\pi\,(n_i - n_j)/q)$$

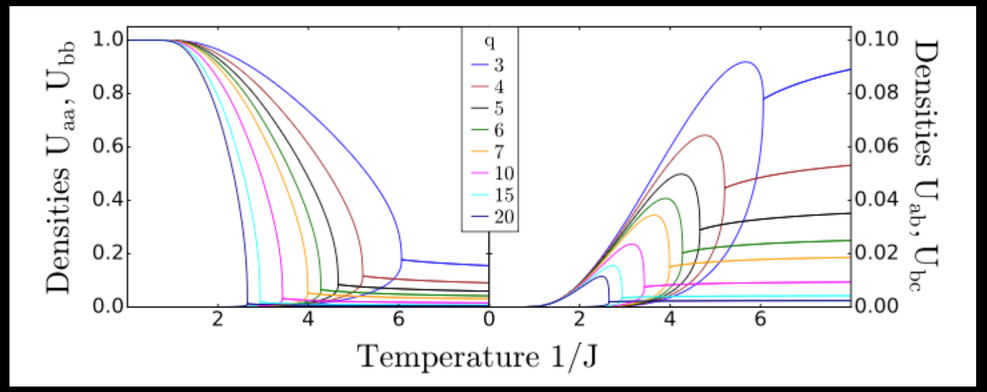
DENSITY RECURSION RELATIONS

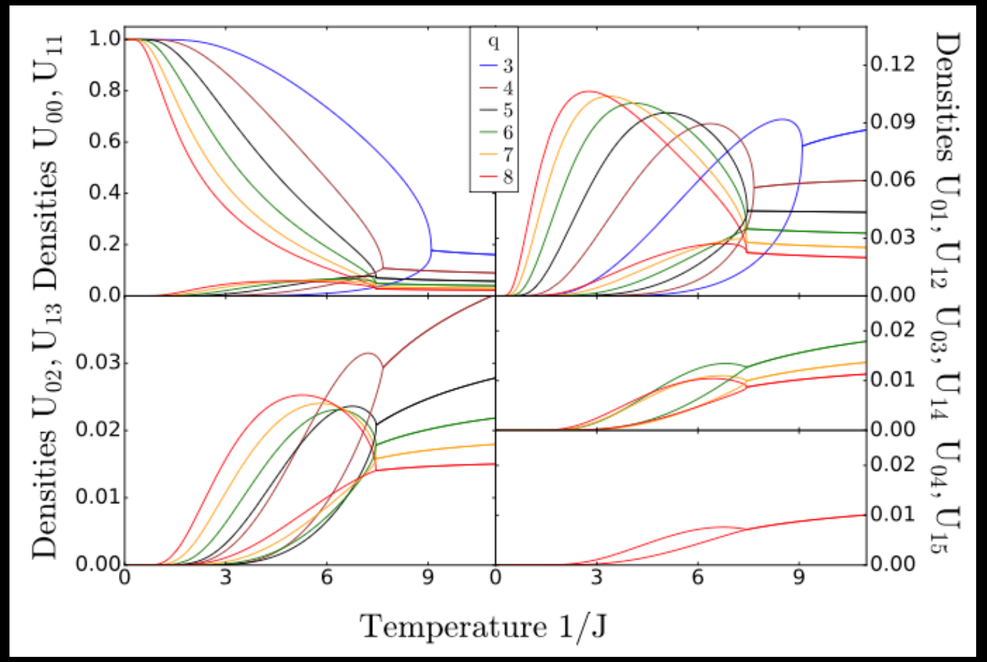
$$\mathbf{U} = b^{-d}\mathbf{U}' \cdot \mathbf{R}$$

$$\mathbf{R} = \partial \mathbf{E}'/\partial \mathbf{E}$$

$$G' = b^d G + \tilde{G}$$

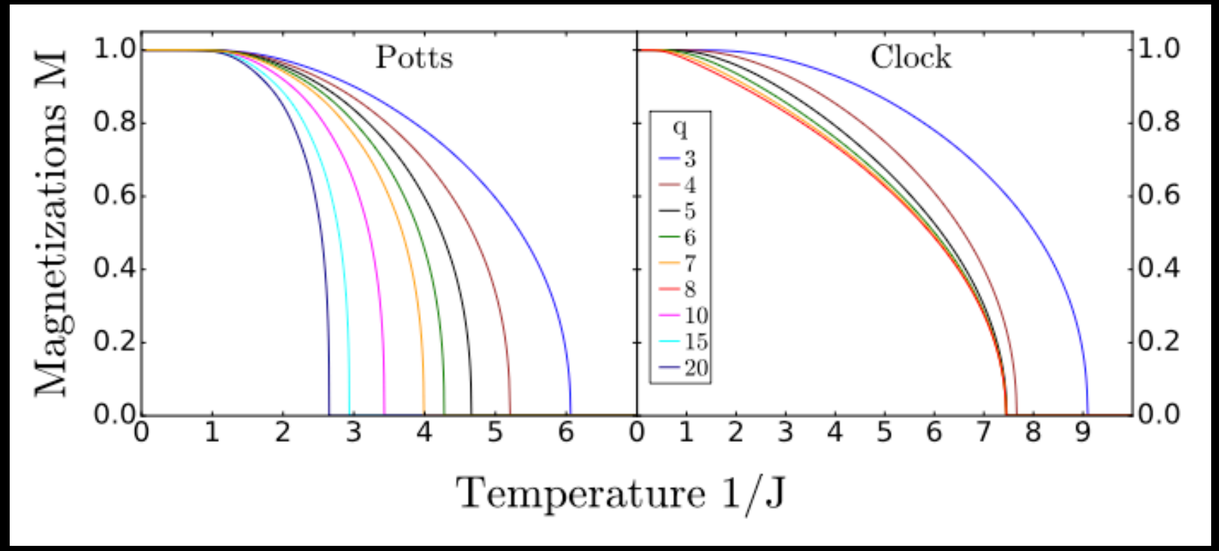
The reentrant behavior is driven by the competition between domain growth and domain elimination during the process of spontaneous symmetry breaking





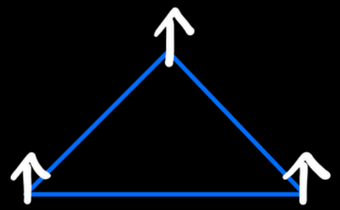
$$M = \langle \delta(s_i, a) \rangle = \sum_{m=0,n=0}^{q-1,q-1} U(mn)\delta(m, a)$$

$$M = \langle \cos(\theta_i) \rangle = \sum_{m=0,n=0}^{q-1,q-1} U(mn)\cos(2\pi m/q)$$

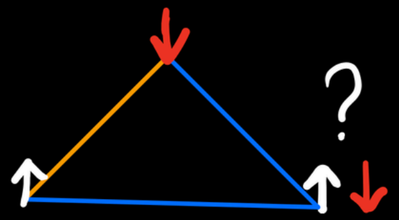


Frustration

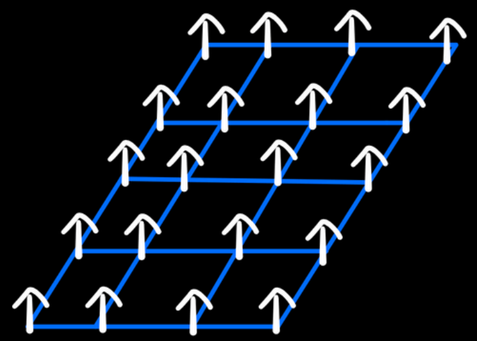
non-frustrated configuration



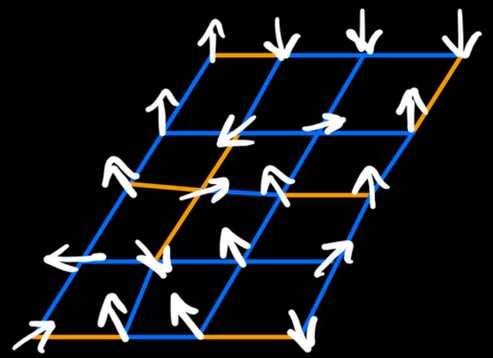
frustrated configuration



ferromagnet



spin glass



ferromagnetic-type bonds

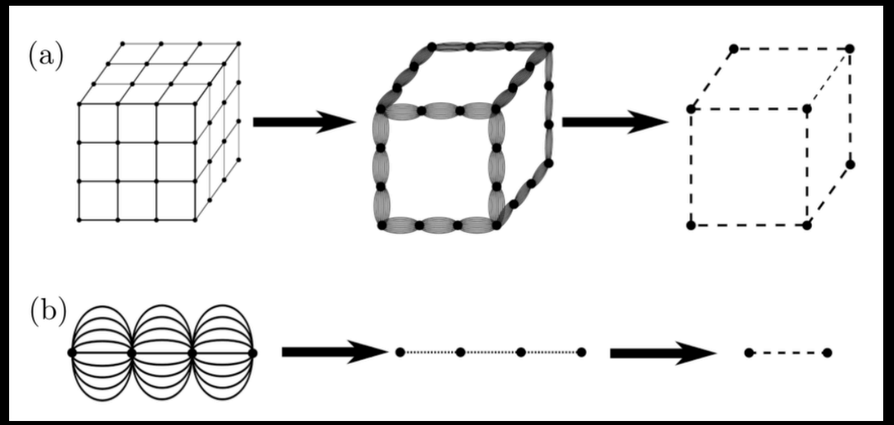
antiferromagnetic-type bonds

Frustration

SPIN-s ISING MODEL

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} J_{ij}(s_i/s)(s_j/s)$$

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} E_{ij}(s_i, s_j)$$



TRANSFER MATRIX IMPLEMENTATION

$$T(s_i, s_j) = e^{E(s_i, s_j)}$$

(bond-moving)

$$\tilde{T}(s_i, s_j) = \prod_{k=1}^{9} T_k(s_i, s_j)$$

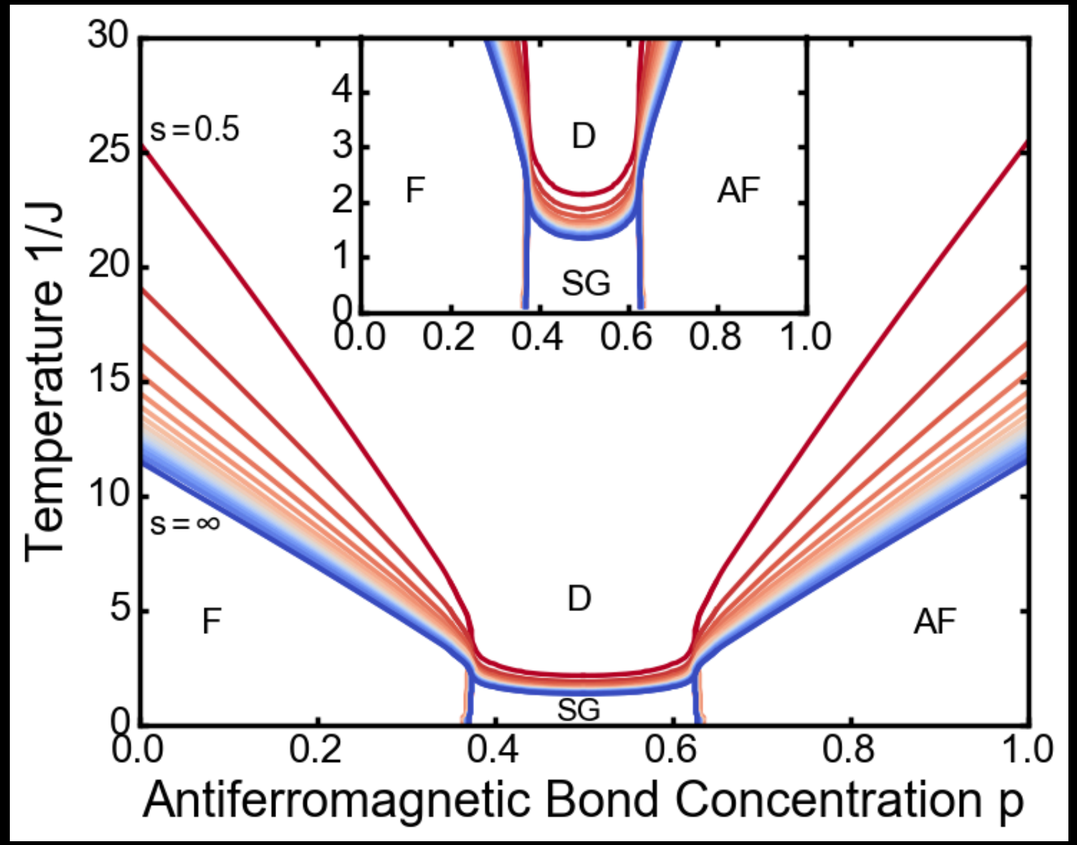
(decimation)

$$\mathbf{T}' = \tilde{\mathbf{T}}_1 \cdot \tilde{\mathbf{T}}_2 \cdot \tilde{\mathbf{T}}_3$$

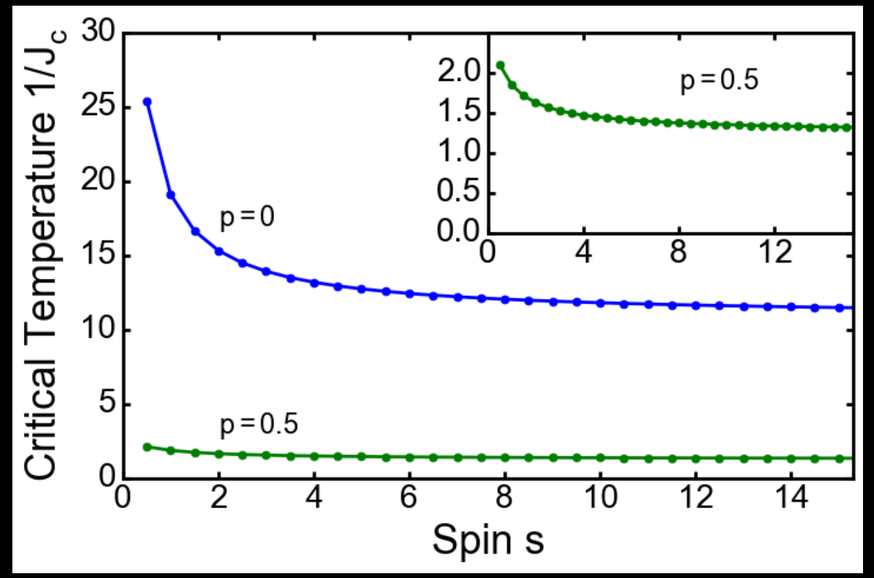
Universal Spin-Glass Chaos

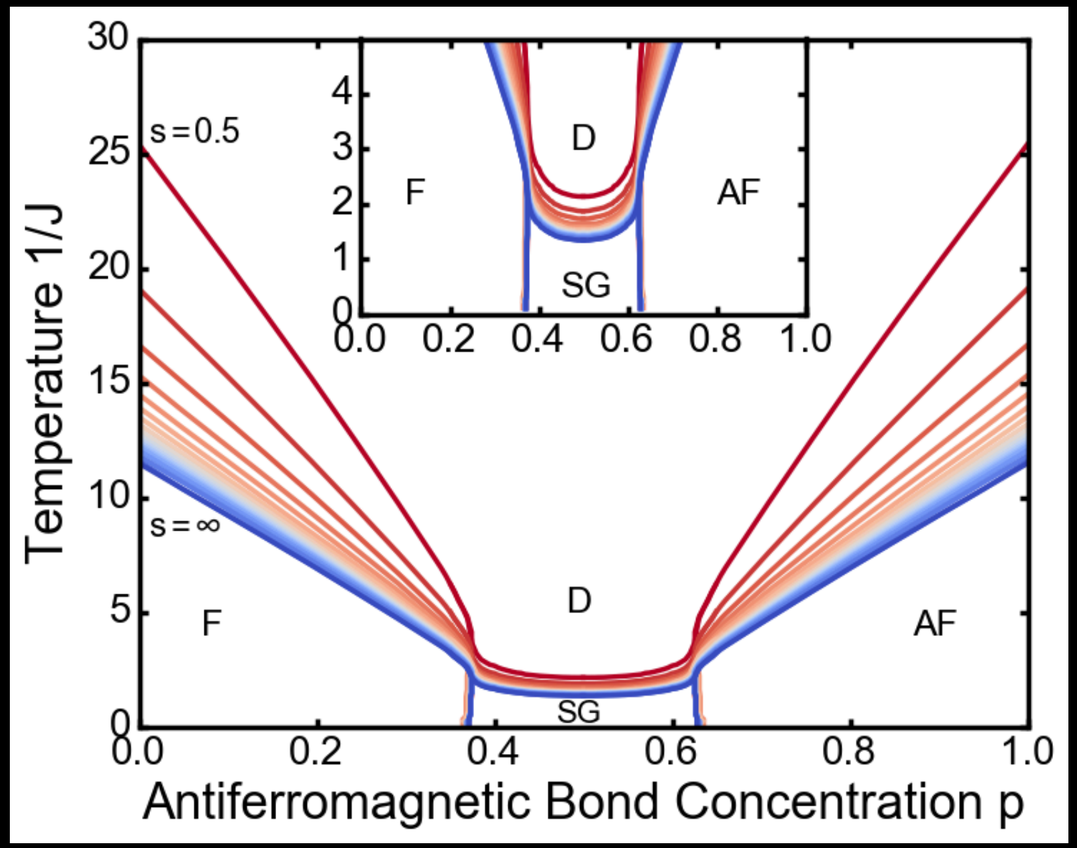
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$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} J_{ij}(s_i/s)(s_j/s)$$



Universal Spin-Glass Chaos



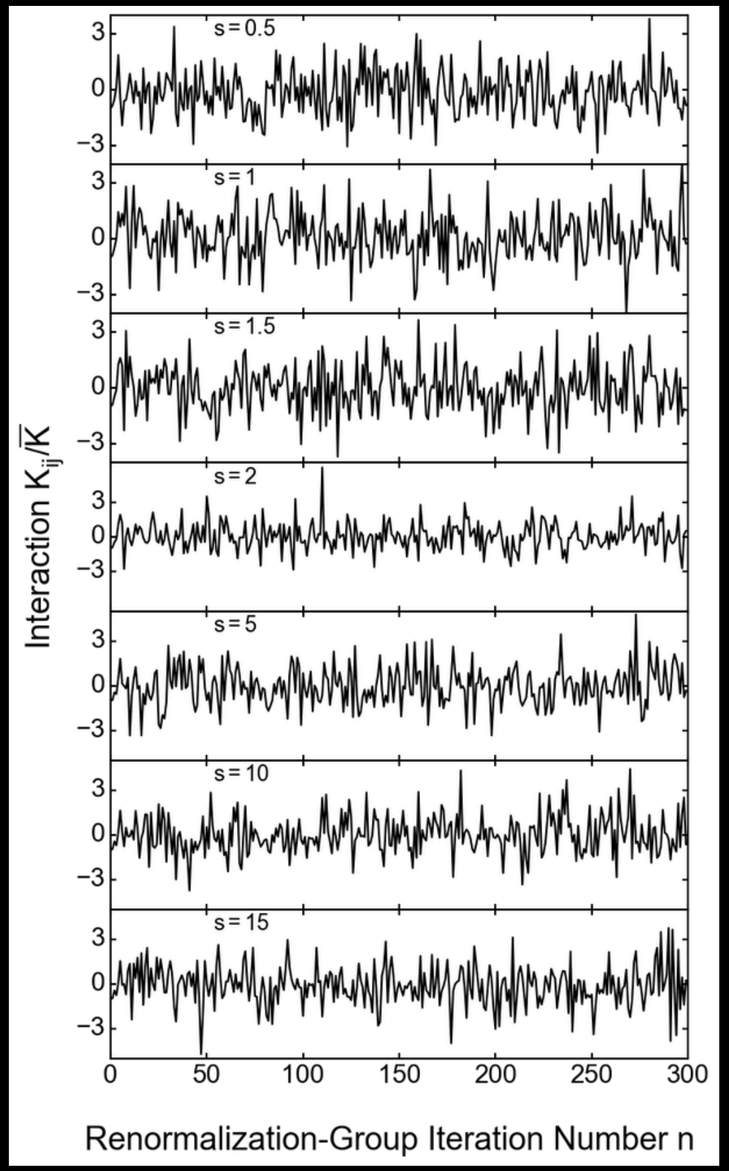


Universal Spin-Glass Chaos

LYAPUNOV EXPONENT

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \ln \left| \frac{dx_{k+1}}{dx_k} \right|$$

$$x_k = K_{ij}/\overline{K}$$



$\lambda = 1.93$ $y_R = 0.24$

RUNAWAY EXPONENT

$$\overline{K'} = b^{y_R}\overline{K}$$

Emergent Nematic Phase

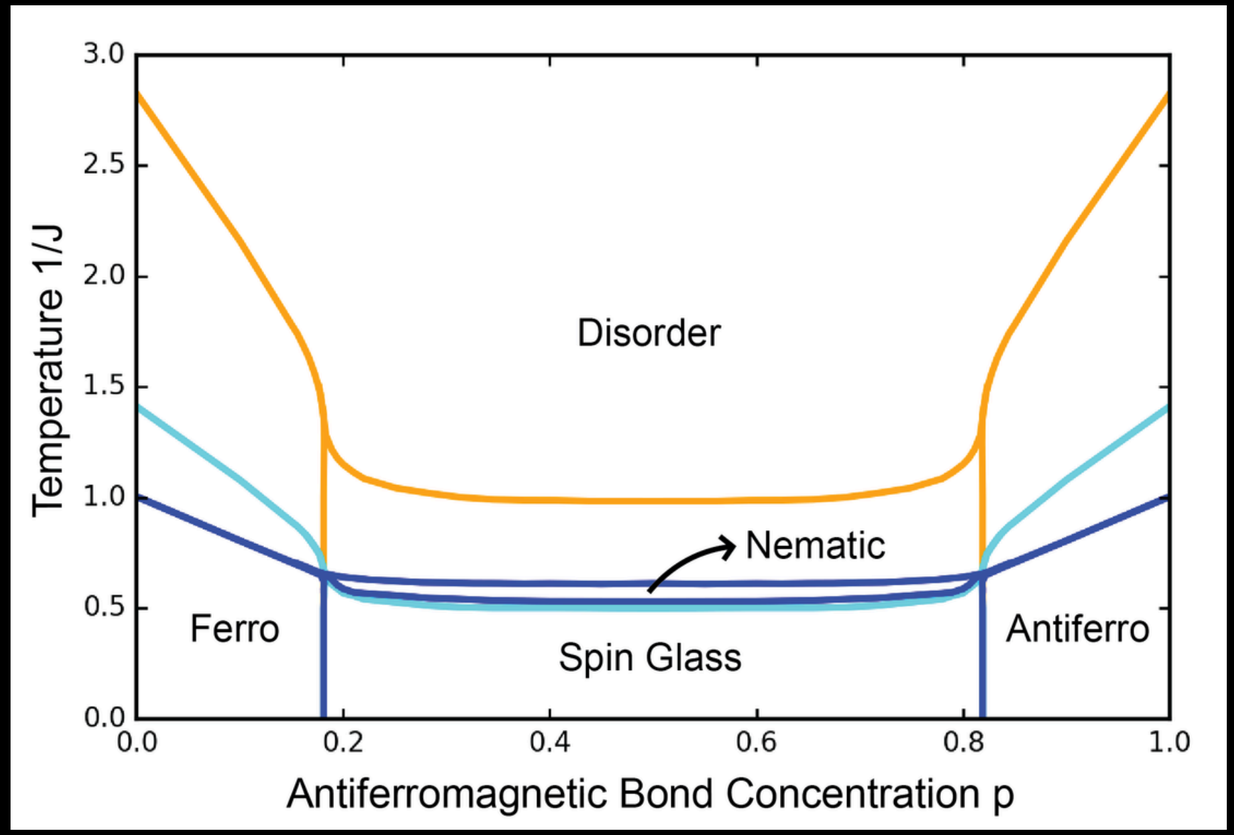
***n*-COMPONENT CUBIC-SPIN SPIN-GLASS SYSTEM**

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} J_{ij}\vec{s}_i \cdot \vec{s}_j \equiv \sum_{\langle ij \rangle} -\beta H_{ij}$$

$$\mathbf{T}_{ij} \equiv e^{-\beta H_{ij}} = e^{J\vec{s}_i \cdot \vec{s}_j}$$

$n = 3$:

$$\mathbf{T}_{ij} = \begin{pmatrix} e^{J} & e^{-J} & 1 & 1 & 1 & 1 \\ e^{-J} & e^{J} & 1 & 1 & 1 & 1 \\ 1 & 1 & e^{J} & e^{-J} & 1 & 1 \\ 1 & 1 & e^{-J} & e^{J} & 1 & 1 \\ 1 & 1 & 1 & 1 & e^{J} & e^{-J} \\ 1 & 1 & 1 & 1 & e^{-J} & e^{J} \end{pmatrix}$$



Emergent Nematic Phase

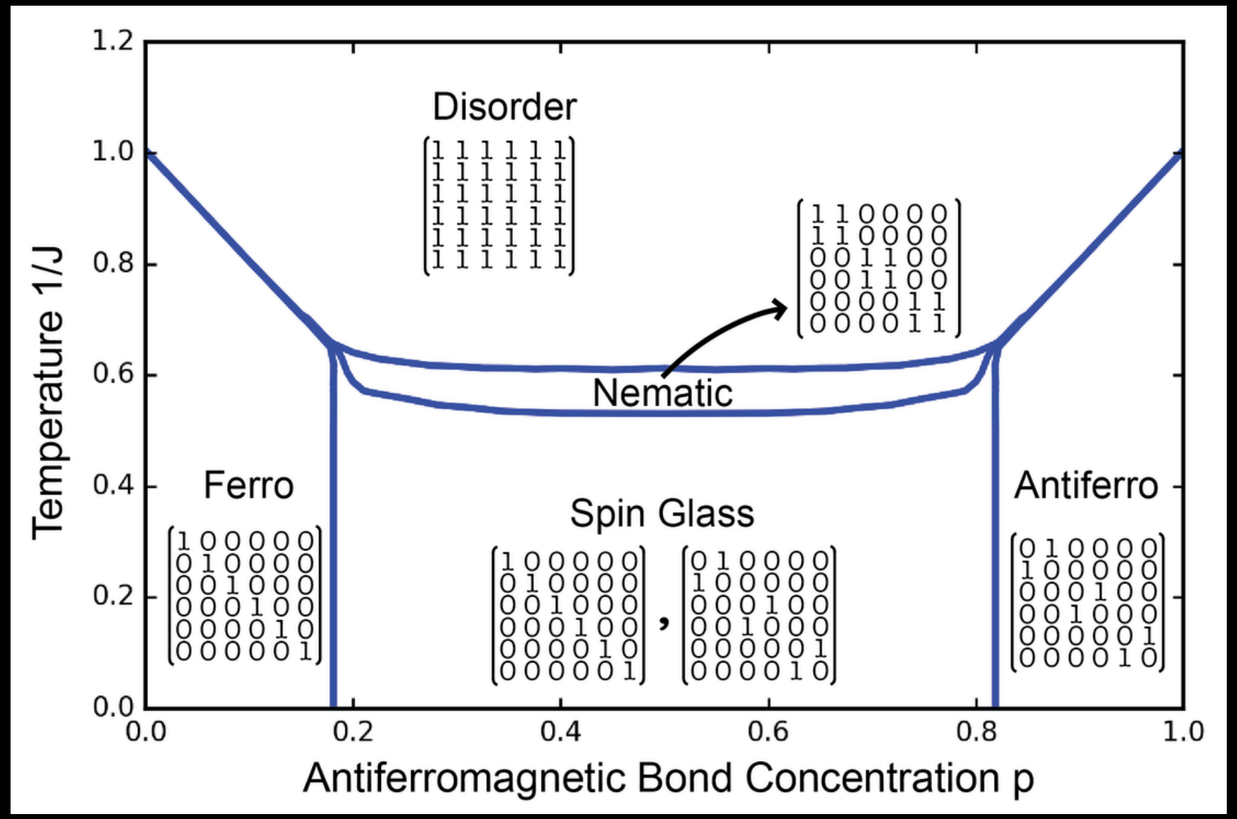
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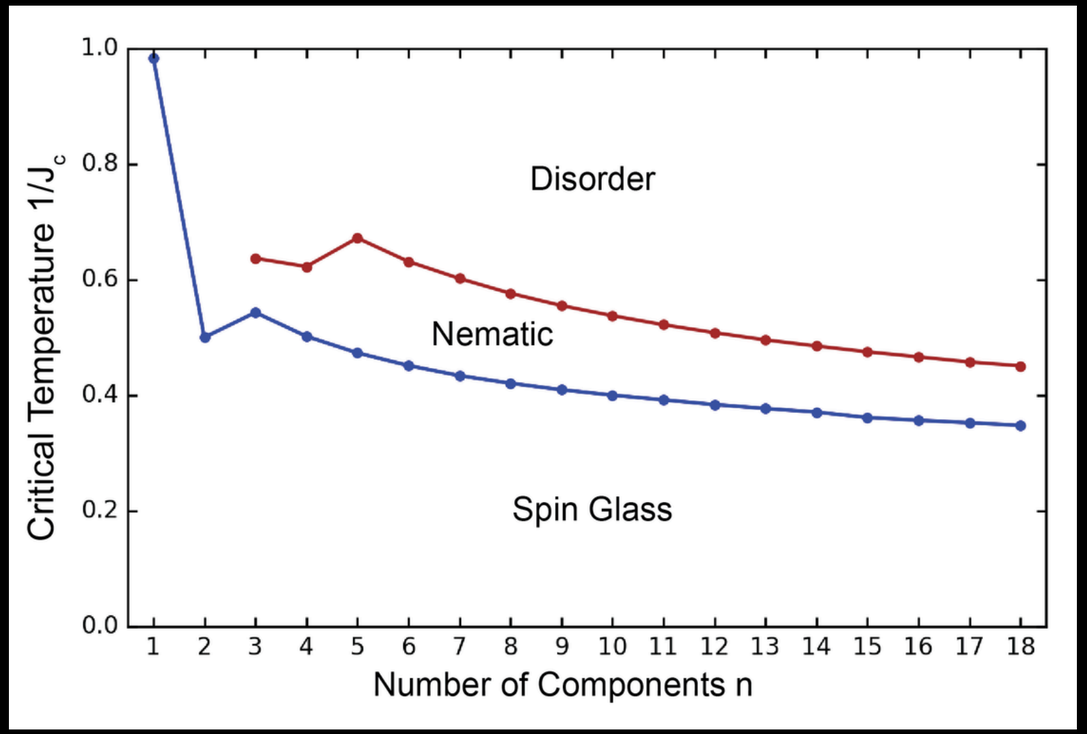
$$\mathbf{T}_{ij} \equiv e^{-\beta H_{ij}} = e^{J\vec{s}_i \cdot \vec{s}_j}$$

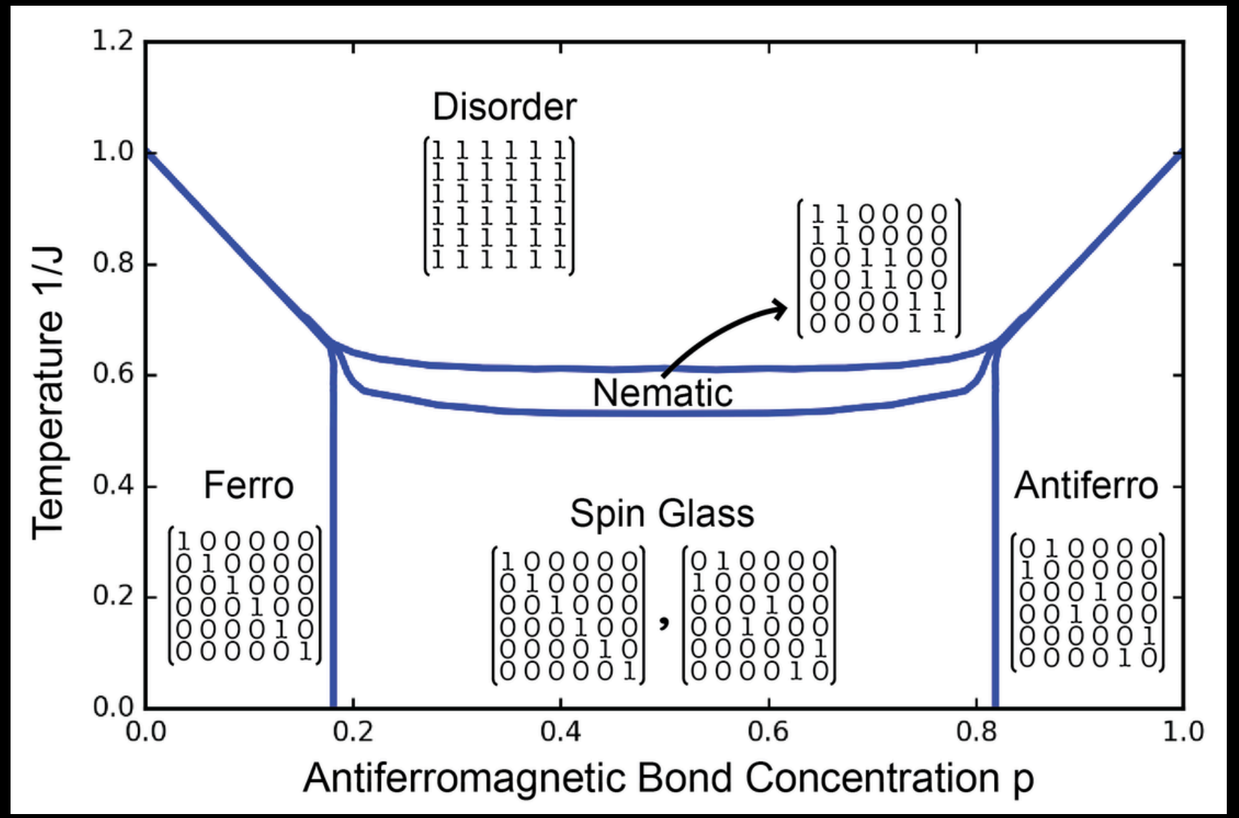
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Emergent Nematic Phase

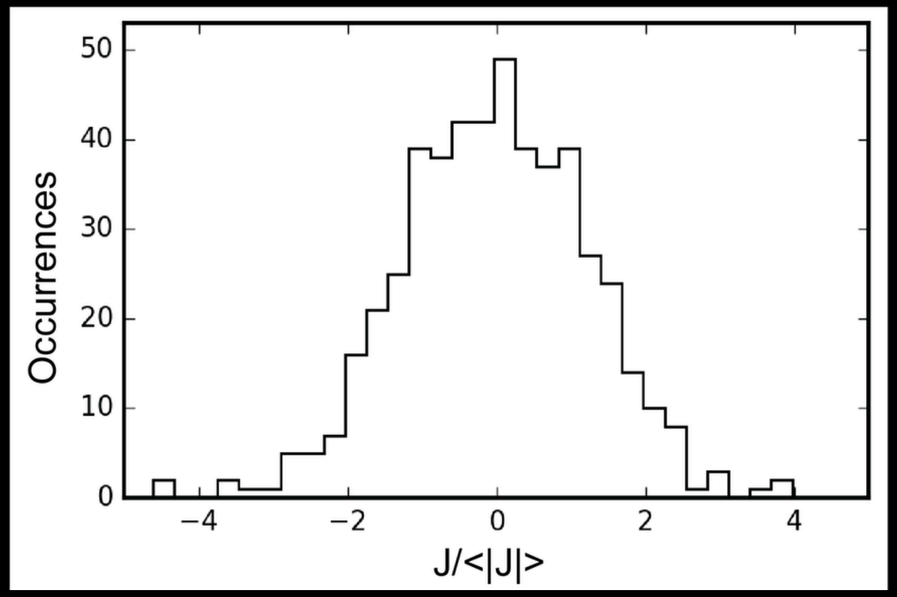


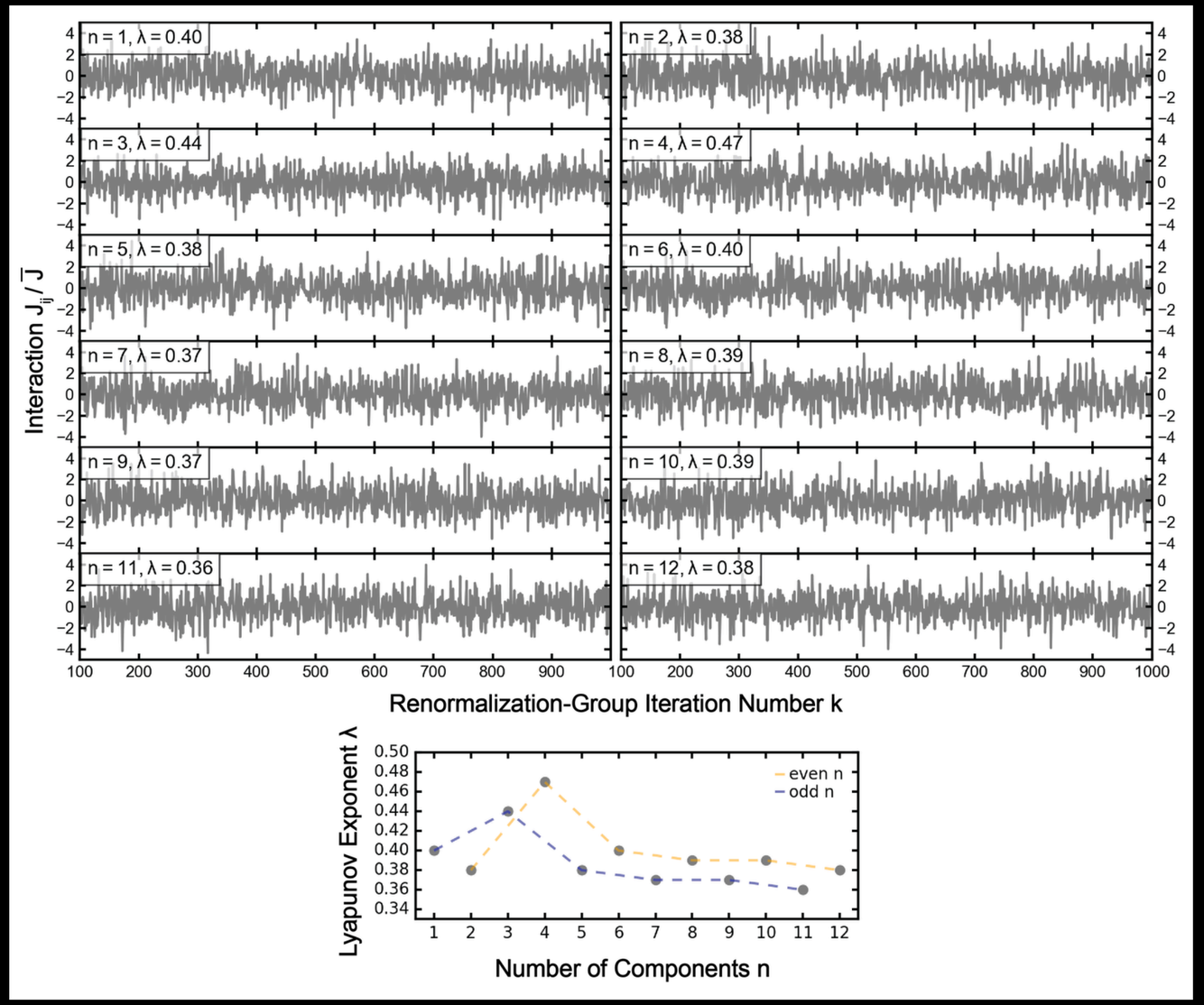


SPIN-GLASS CHAOS

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \ln \left| \frac{dx_{k+1}}{dx_k} \right|,$$

$$x_k = J_{ij}/\overline{J}$$





Applying External Magnetic Fields

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} \left[J_{ij}\vec{s}_i \cdot \vec{s}_j + \vec{H} \cdot (\vec{s}_i + \vec{s}_j) \right] \equiv \sum_{\langle ij \rangle} -\beta H_{ij}$$

$$\mathbf{T}_{ij} \equiv e^{-\beta H_{ij}} = e^{J\vec{s}_i \cdot \vec{s}_j + \vec{H} \cdot (\vec{s}_i + \vec{s}_j)}$$

AXIAL FIELD

$$\mathbf{T}_{ij} = \begin{pmatrix} e^{J+2H} & e^{-J} & e^{H} & e^{H} & e^{H} & e^{H} \\ e^{-J} & e^{J-2H} & e^{-H} & e^{-H} & e^{-H} & e^{-H} \\ e^{H} & e^{-H} & e^{J} & e^{-J} & 1 & 1 \\ e^{H} & e^{-H} & e^{-J} & e^{J} & 1 & 1 \\ e^{H} & e^{-H} & 1 & 1 & e^{J} & e^{-J} \\ e^{H} & e^{-H} & 1 & 1 & e^{-J} & e^{J} \end{pmatrix}$$

PLANAR-DIAGONAL FIELD

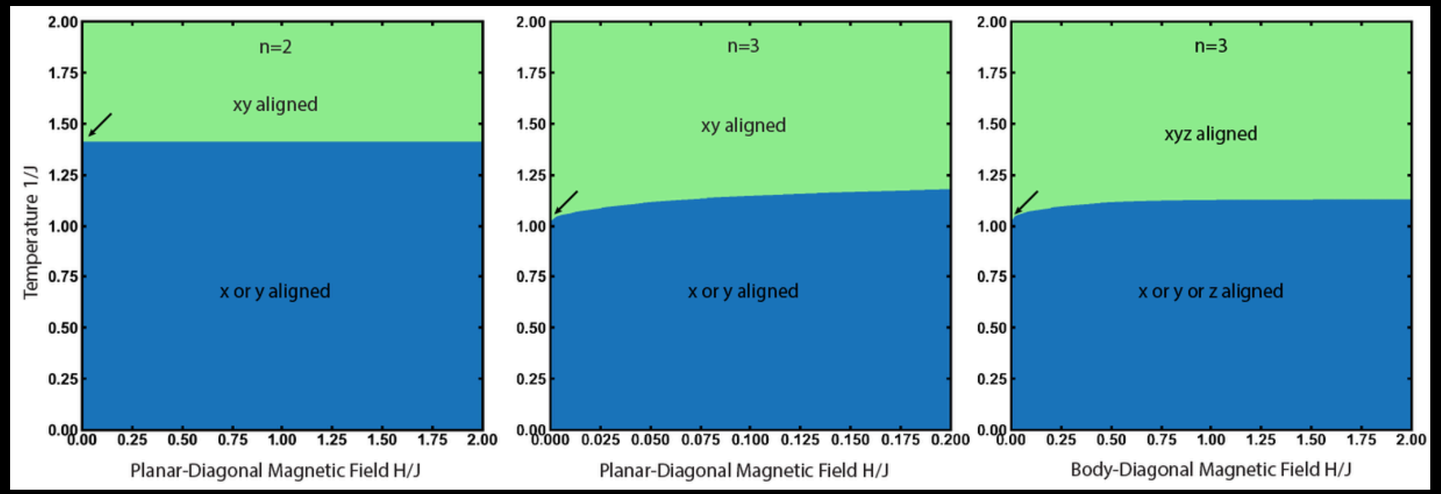
$$\mathbf{T}_{ij} = \begin{pmatrix} e^{J+2H} & e^{-J} & e^{H} & e^{H} & e^{H} & e^{H} \\ e^{-J} & e^{J-2H} & e^{-H} & e^{-H} & e^{-H} & e^{-H} \\ e^{H} & e^{-H} & e^{J} & e^{-J} & 1 & 1 \\ e^{H} & e^{-H} & e^{-J} & e^{J} & 1 & 1 \\ e^{H} & e^{-H} & 1 & 1 & e^{J} & e^{-J} \\ e^{H} & e^{-H} & 1 & 1 & e^{-J} & e^{J} \end{pmatrix}$$

BODY-DIAGONAL FIELD

$$\mathbf{T}_{ij} = \begin{pmatrix} e^{J+2H} & e^{-J} & e^{H} & e^{H} & e^{H} & e^{H} \\ e^{-J} & e^{J-2H} & e^{-H} & e^{-H} & e^{-H} & e^{-H} \\ e^{H} & e^{-H} & e^{J} & e^{-J} & 1 & 1 \\ e^{H} & e^{-H} & e^{-J} & e^{J} & 1 & 1 \\ e^{H} & e^{-H} & 1 & 1 & e^{J} & e^{-J} \\ e^{H} & e^{-H} & 1 & 1 & e^{-J} & e^{J} \end{pmatrix}$$

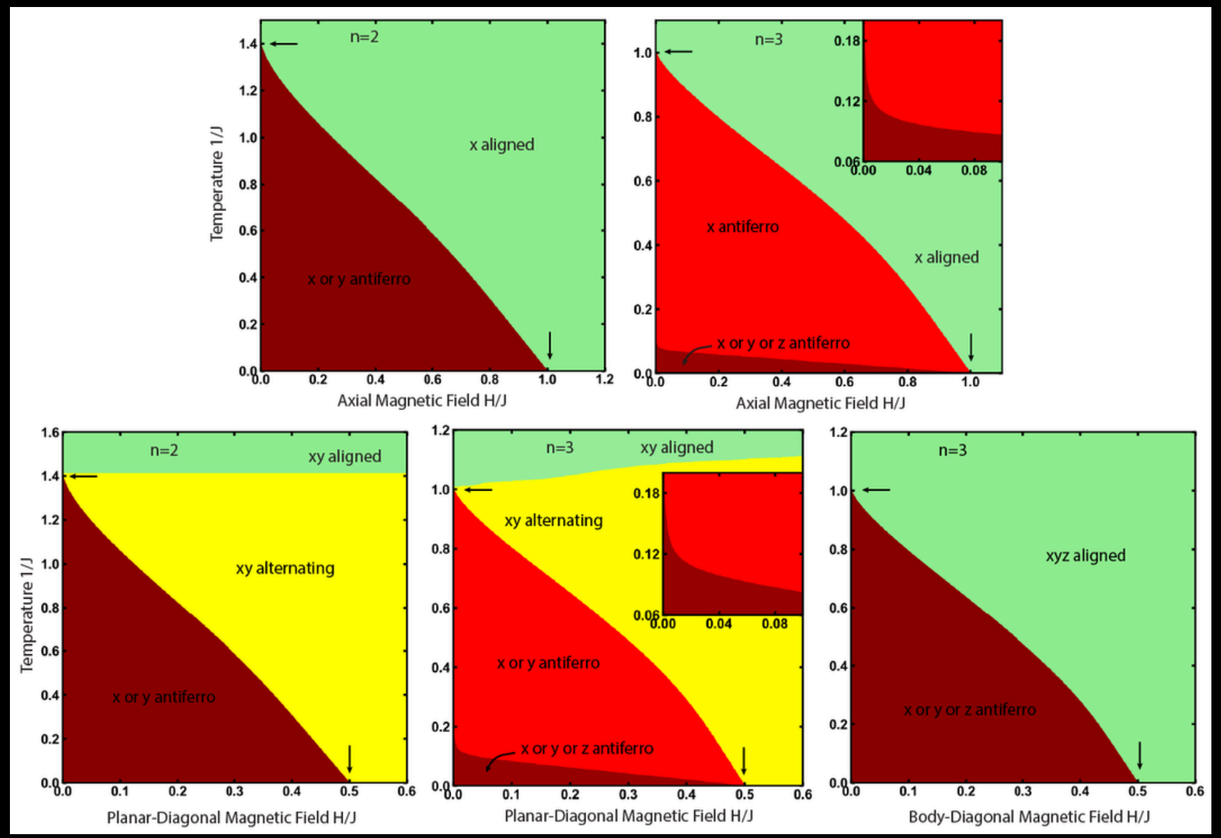
FIELDS ON FERROMAGNETIC SYSTEMS

$p = 0$



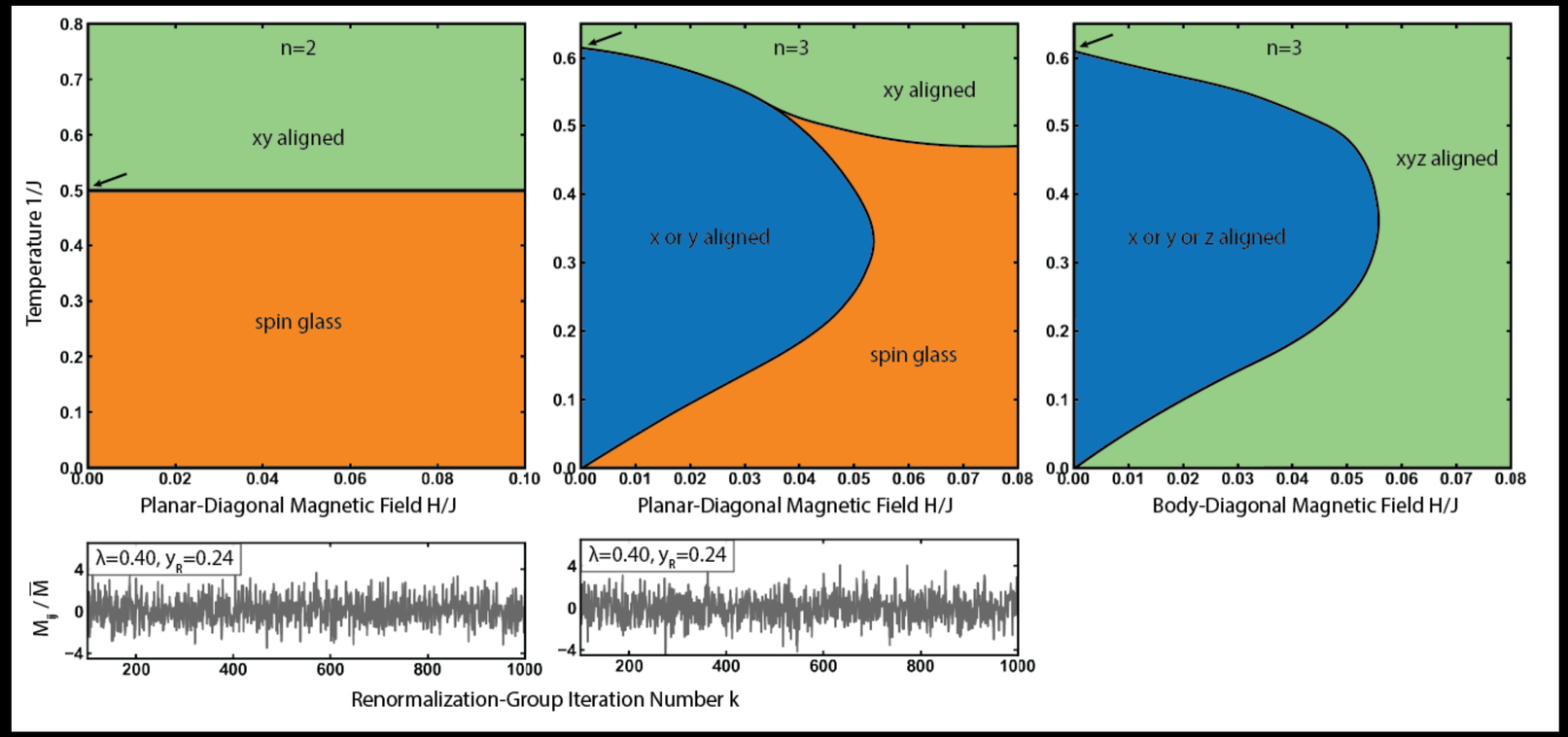
FIELDS ON ANTIFERROMAGNETIC SYSTEMS

$p = 1$



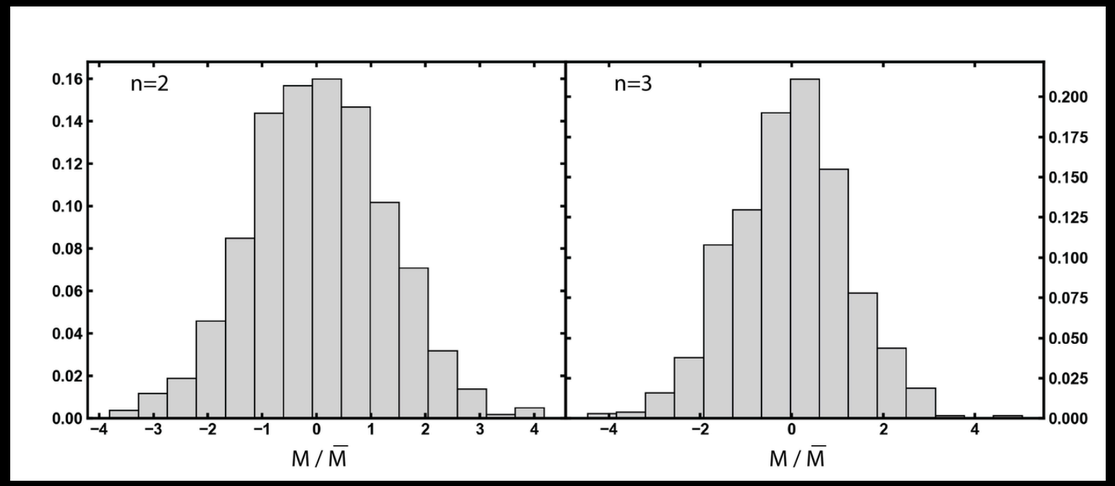
FIELDS ON SPIN-GLASS SYSTEMS

$p = 0.5$



$$M \equiv \frac{1}{2}[-\beta H_{ij}(+x,+x) - \beta H_{ij}(+x,+y)]$$

$$J = \frac{1}{2}[-\beta H_{ij}(+x,+x) - \beta H_{ij}(+x,-x)]$$



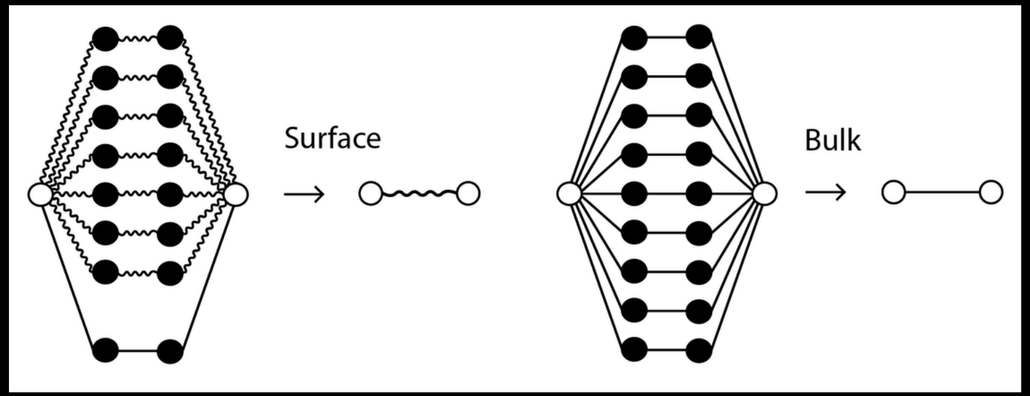
Spin-Glass Sponges

BULK AND SURFACE INTERACTIONS

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle}^{S} J_{ij}^{S} \boldsymbol{s}_i \boldsymbol{s}_j + \sum_{\langle ij \rangle}^{B} J_{ij}^{B} \boldsymbol{s}_i \boldsymbol{s}_j$$

$f$lat surface: $d_S = d - 1 = 2$ for bulk $d = 3$

$f$ractal (sponge) surface: $d - 1 < d_S < d$, i.e., $2 < d_S < 3$



Spin-Glass Sponges

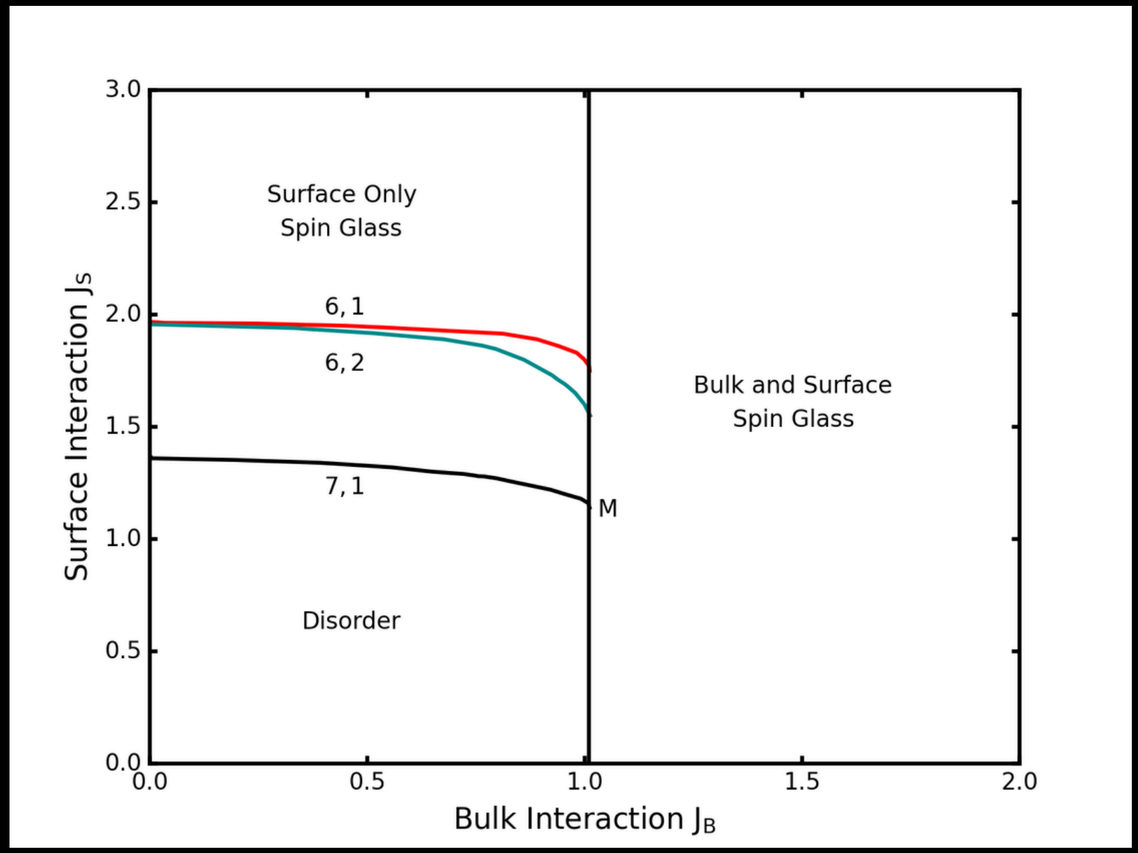
PHASE DIAGRAMS

$d_S = 2.77, 2.63$

(b^{d_S}, n)

$$n = \frac{b^{d-1} - b^{d_S-1}}{2}$$

no surface-only spin-glass phase for $d_S \leq 2.46$

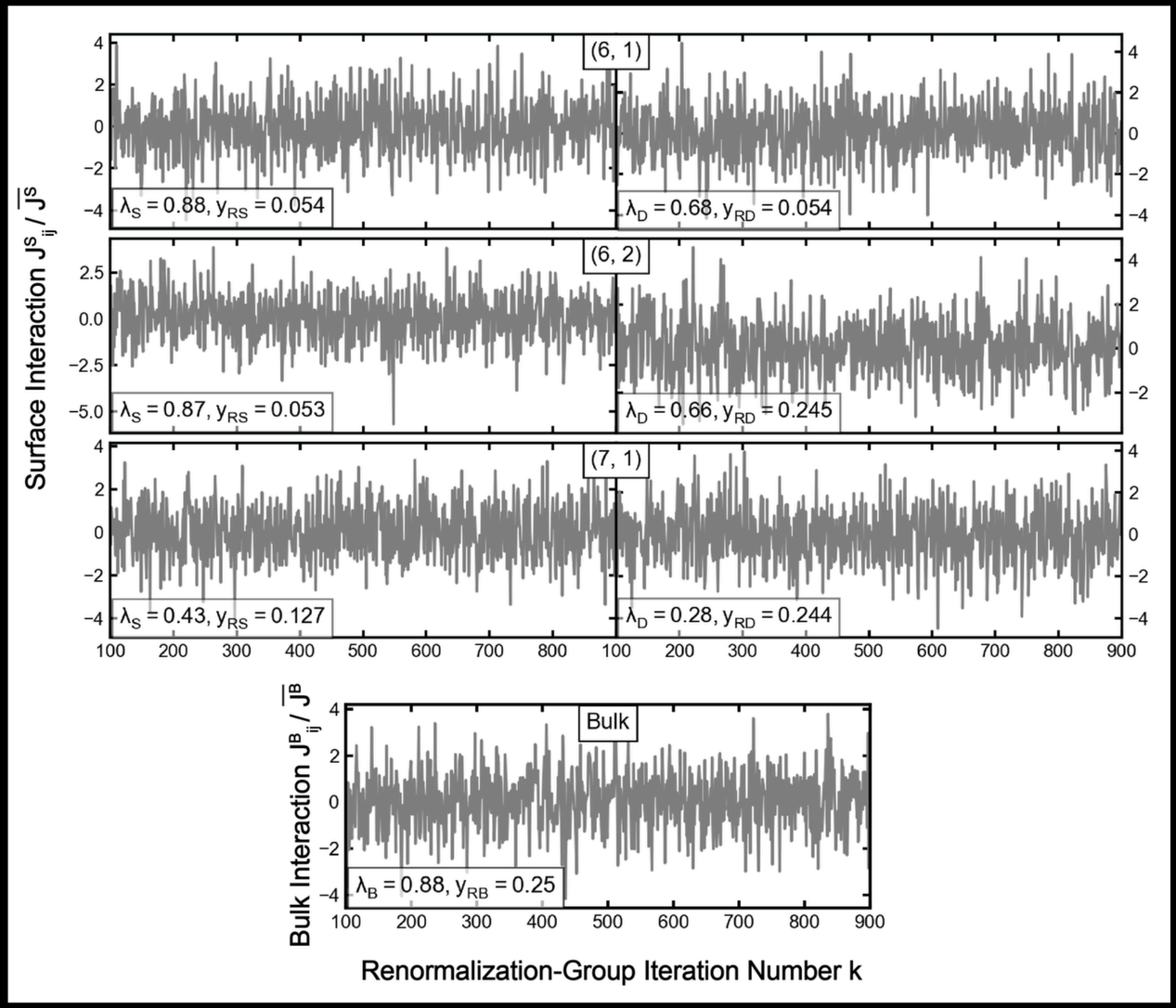


Spin-Glass Sponges

SPIN-GLASS CHAOS

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \ln \left| \frac{dx_{k+1}}{dx_k} \right|$$

$$\overline{J'} = b^{y_R} \overline{J}$$



Symmetry Competition: Potts vs. Clock

MODEL

no "neighboring" states in spin space: all states that differ from a reference state are energetically equivalent

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} \left[J\,\delta(\vec{s}_i, \vec{s}_j) + K\,\vec{s}_i \cdot \vec{s}_j \right]$$

angular proximity matters: states that are "nearby" in angle contribute differently than states that are "far apart"

$$\theta_i = 2\pi n_i / q$$
$$n_i = 0, 1, \ldots, q-1$$

Symmetry Competition: Potts vs. Clock

TRANSFER MATRIX

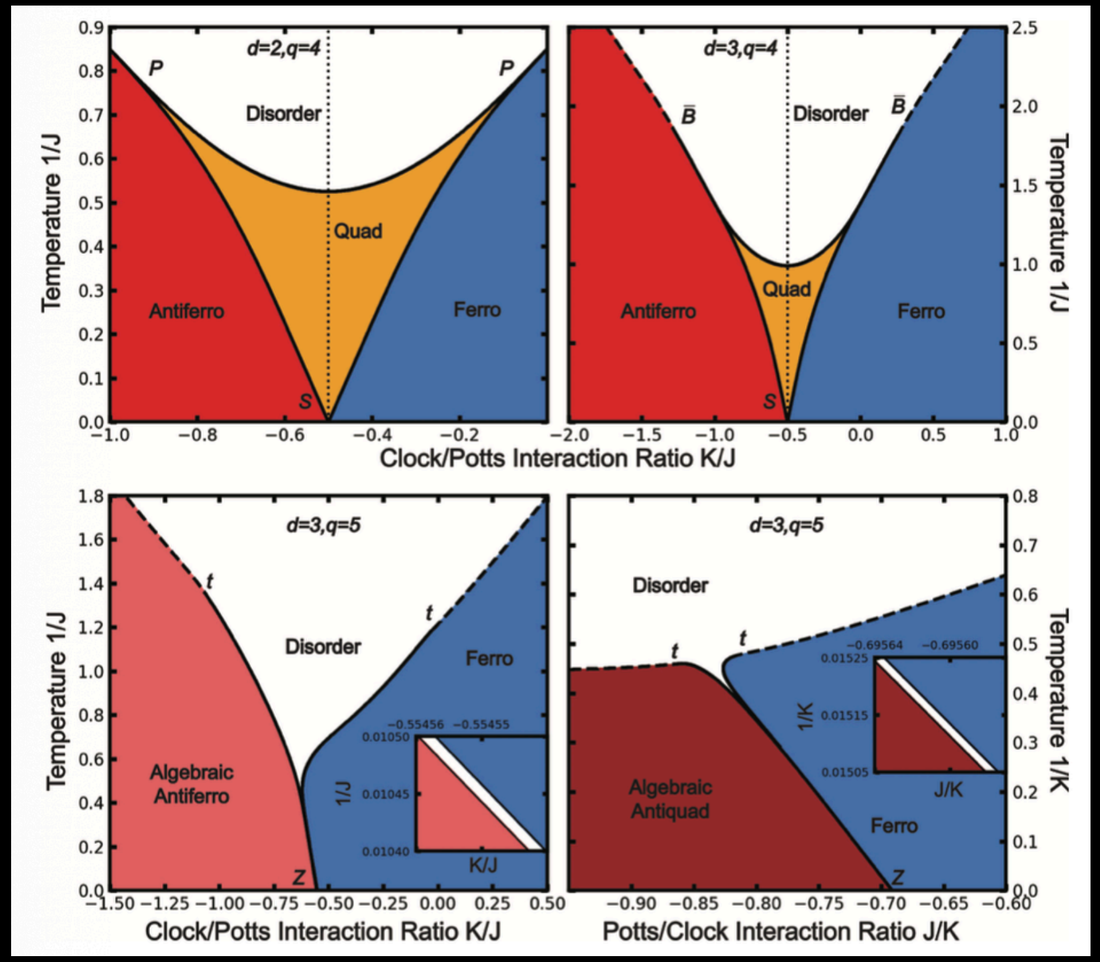
$$\mathbf{T}_{ij} \equiv e^{-\beta H_{ij}} = \begin{pmatrix} e^{J+K} & 1 & e^{-K} & 1 \\ 1 & e^{J+K} & 1 & e^{-K} \\ e^{-K} & 1 & e^{J+K} & 1 \\ 1 & e^{-K} & 1 & e^{J+K} \end{pmatrix}$$

$$\mathbf{T}_{ij} \equiv e^{-\beta H_{ij}} = \begin{pmatrix} e^{J+K} & e^{0.31K} & e^{-0.81K} & e^{-0.81K} & e^{0.31K} \\ e^{0.31K} & e^{J+K} & e^{0.31K} & e^{-0.81K} & e^{-0.81K} \\ e^{-0.81K} & e^{0.31K} & e^{J+K} & e^{0.31K} & e^{-0.81K} \\ e^{-0.81K} & e^{-0.81K} & e^{0.31K} & e^{J+K} & e^{0.31K} \\ e^{0.31K} & e^{-0.81K} & e^{-0.81K} & e^{0.31K} & e^{J+K} \end{pmatrix}$$

EFFECTIVE VACANCIES

$$\mathbf{T}_{ij} = \begin{pmatrix} e^{J+K} & 1 & e^{-K} & 1 & (q-1)^{1/2d} \\ 1 & e^{J+K} & 1 & e^{-K} & (q-1)^{1/2d} \\ e^{-K} & 1 & e^{J+K} & 1 & (q-1)^{1/2d} \\ 1 & e^{-K} & 1 & e^{J+K} & (q-1)^{1/2d} \\ (q-1)^{1/2d} & (q-1)^{1/2d} & (q-1)^{1/2d} & (q-1)^{1/2d} & (q-1)^{1/d} \end{pmatrix}$$

the competition is a struggle between the permutation-driven "sameness" of the Potts model and the angle-driven "closeness" of the clock model



Continuous Spin States: XY-Ashkin-Teller Model

XY model

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} J\,\vec{s}_i \cdot \vec{s}_j$$

Ashkin-Teller model

$$-\beta\mathcal{H} = J_1 \sum_{\langle ij \rangle} s_i s_j + J_2 \sum_{\langle ij \rangle} s_i s_j + M \sum_{\langle ij \rangle} s_i s_j t_i t_j$$

$$-\beta\mathcal{H} = \sum_{\langle ij \rangle} \left[J \left(\vec{s}_i \cdot \vec{s}_j + \vec{t}_i \cdot \vec{t}_j \right) + M \left(\vec{s}_i \cdot \vec{s}_j \right) \left(\vec{t}_i \cdot \vec{t}_j \right) \right]$$

$$= \sum_{\langle ij \rangle} -\beta H_{ij}\left(\vec{s}_i, \vec{t}_i; \vec{s}_j, \vec{t}_j \right)$$

Ashkin-Tellerized *XY* model

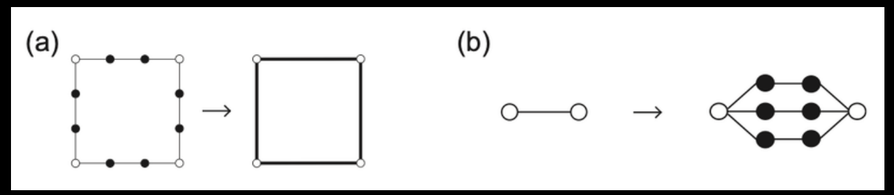
RG Approach

DOUBLE FOURIER REPRESENTATION

$$u_{ij}(\theta_{ij}, \phi_{ij}) = e^{-\beta \mathcal{H}_{ij}(\vec{s}_i, \vec{t}_i; \vec{s}_j, \vec{t}_j)}$$

RG Approach

DOUBLE FOURIER REPRESENTATION



DECIMATION

$$\tilde{u}_{13}(\theta_{13},\phi_{13}) = \int_0^{2\pi} u_{12}(\theta_{12},\phi_{12}) u_{23}(\theta_{23},\phi_{23}) \frac{d\theta_2}{2\pi}\frac{d\phi_2}{2\pi}$$

$$f(k,l) = \int_0^{2\pi} u(\theta,\phi) e^{ik\theta+il\phi} \frac{d\theta}{2\pi}\frac{d\phi}{2\pi}$$

$$u(\theta,\phi) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} e^{-ik\theta-il\phi} f(k,l)$$

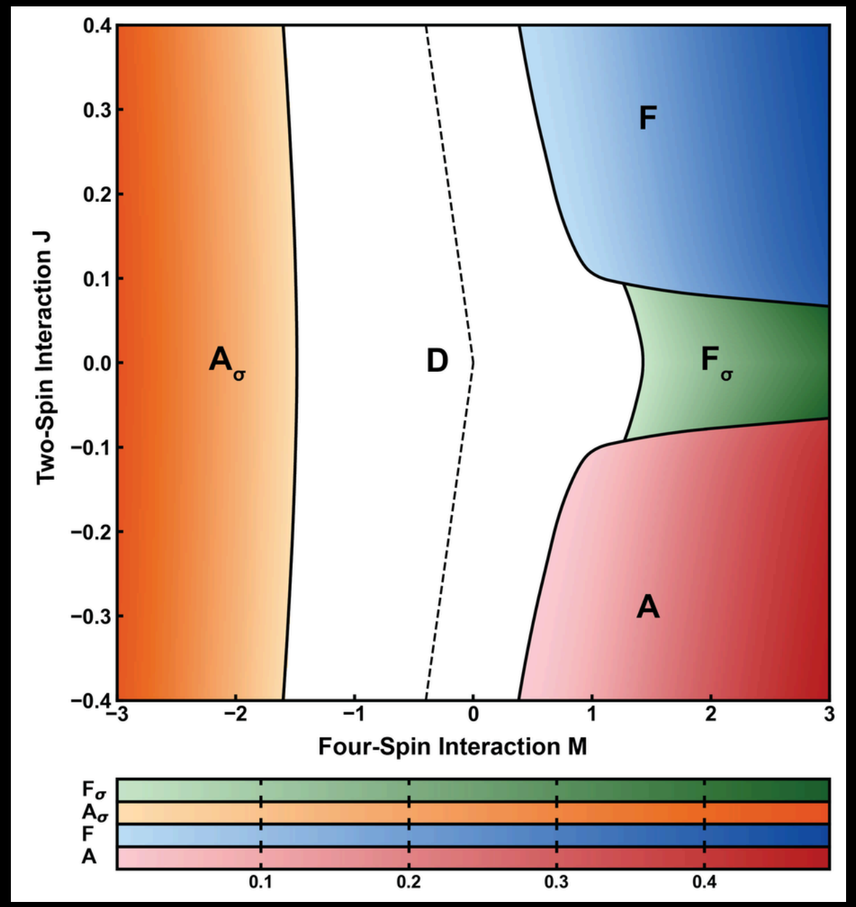
$$\tilde{f}_{13}(k,l) = f_{12}(k,l) f_{23}(k,l)$$

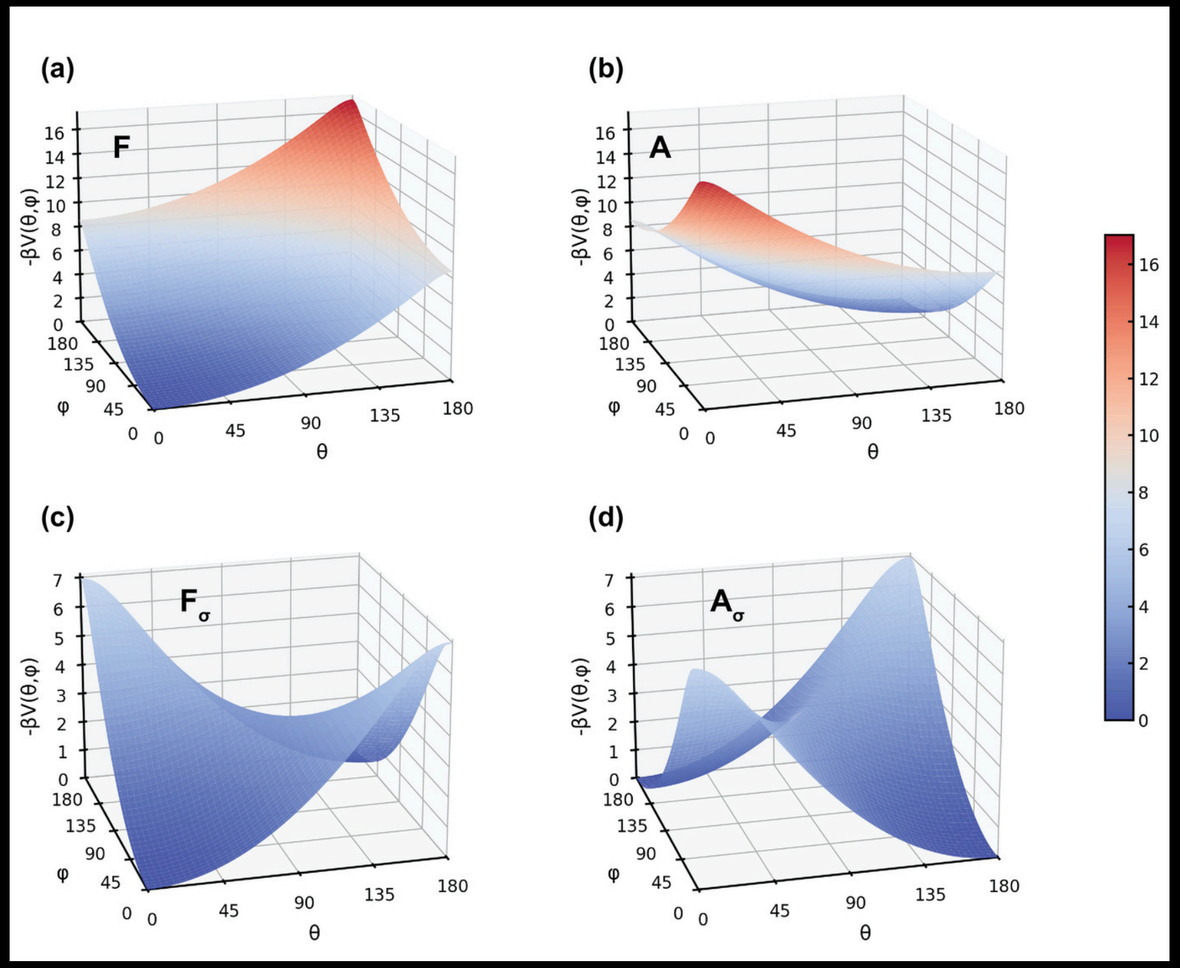
BOND-MOVING

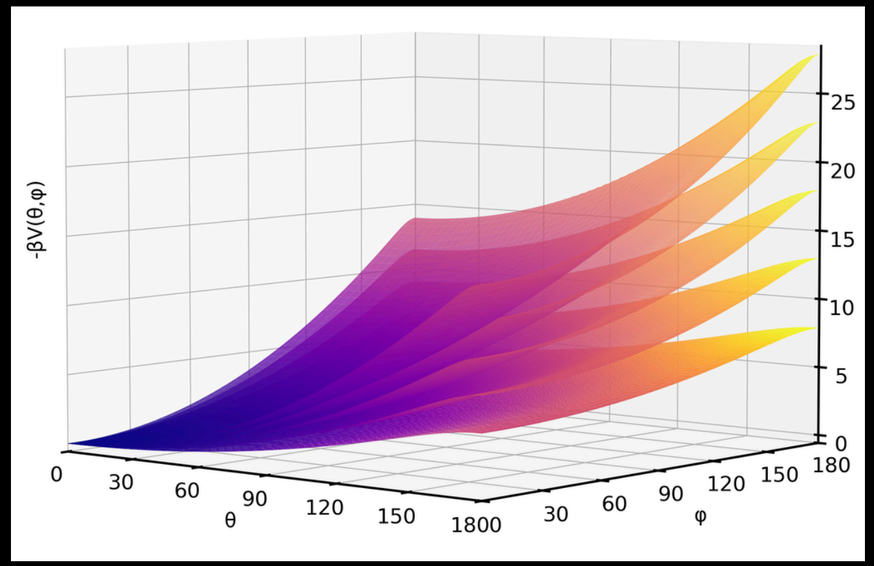
$$u'_{ij}(\theta,\phi) = \tilde{u}_{i_1 j_1}(\theta,\phi)\tilde{u}_{i_2 j_2}(\theta,\phi)$$

$$f'_{ij}(k,l) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \tilde{f}_{i_1 j_1}(k-m,l-n)\tilde{f}_{i_2 j_2}(m,n)$$

- F : $\vec{s}$ and $\vec{t}$ spins order ferromagnetically
- A : $\vec{s}$ and $\vec{t}$ spins order antiferromagnetically
- F_σ : the composite variable $\vec{s}\cdot\vec{t}$ orders ferromagnetically
- A_σ : the composite variable orders antiferromagnetically
- D : no long-range order




(a)
F
(b)
A
(c)
F_σ
(d)
A_σ
-βV(θ,φ)
θ
φ
0
45
90
135
180
2
4
6
8
10
12
14
16
1
3
5
7

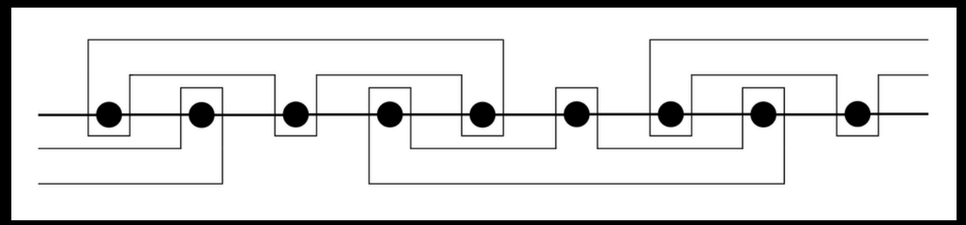

-βV(θ,φ)
θ
φ
0
30
60
90
120
150
1800
5
10
15
20
25

Long-Range Interactions

ISING MODEL

$$-\beta\mathcal{H} = \sum_{r_1 \neq r_2} J|r_1 - r_2|^{-a} s_{r_1} s_{r_2} + H \sum_{r_1} s_{r_1}$$

NIEMEYER-VAN LEEUWEN CELL-SPIN CONSTRUCTION



$$s'_{r'} = \text{signum}(s_{r-4} + s_{r-2} + s_r)$$

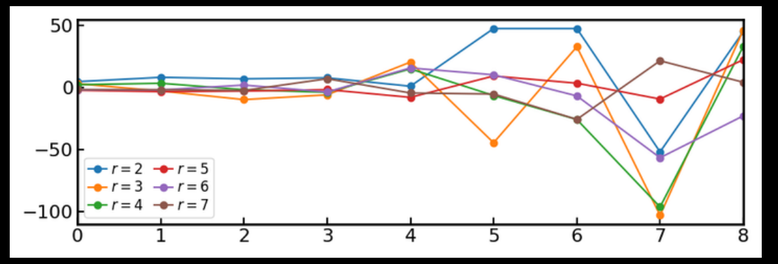
RG: RECURSION RELATIONS

$$J'_{r'} = \frac{1}{4} \ln \frac{R_{r'}(+1,+1) R_{r'}(-1,-1)}{R_{r'}(+1,-1) R_{r'}(-1,+1)},$$

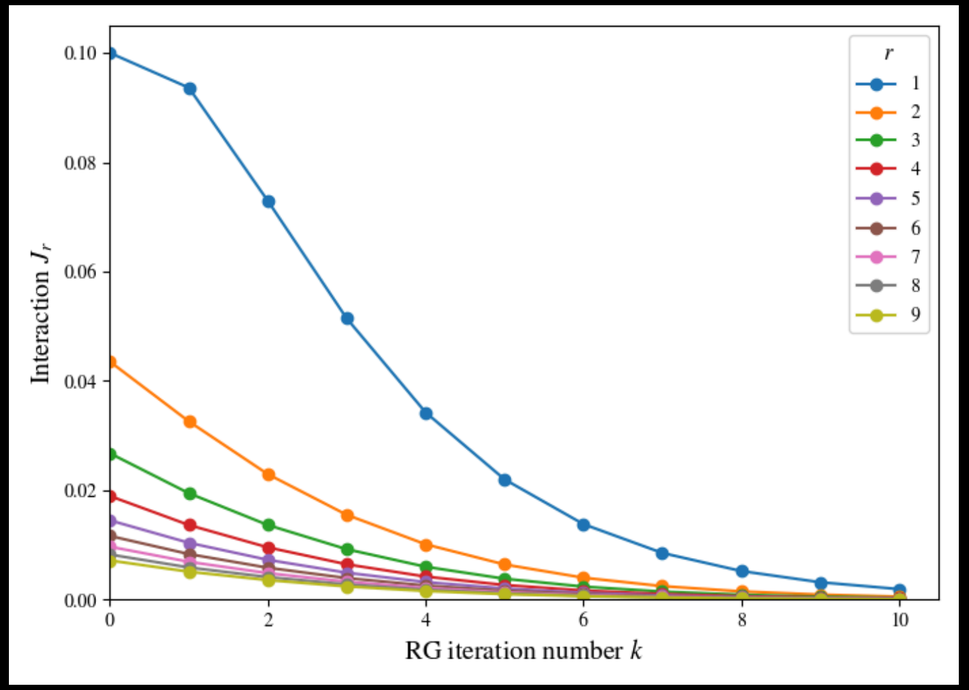
$$H' = \frac{1}{4} \ln \frac{R_1(+1,+1)}{R_1(-1,-1)},$$

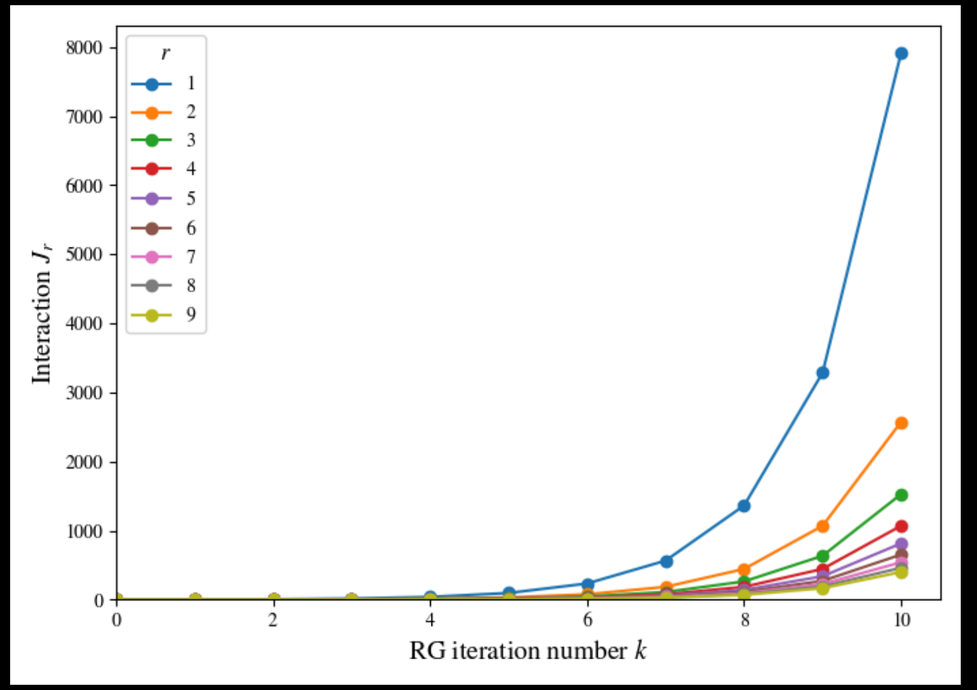
$$R_{r'}(s'_0, s'_{r'}) = \sum_{\sigma_0, \sigma_{r'}} e^{-\beta H_{0r'}}$$

Long-Range Interactions



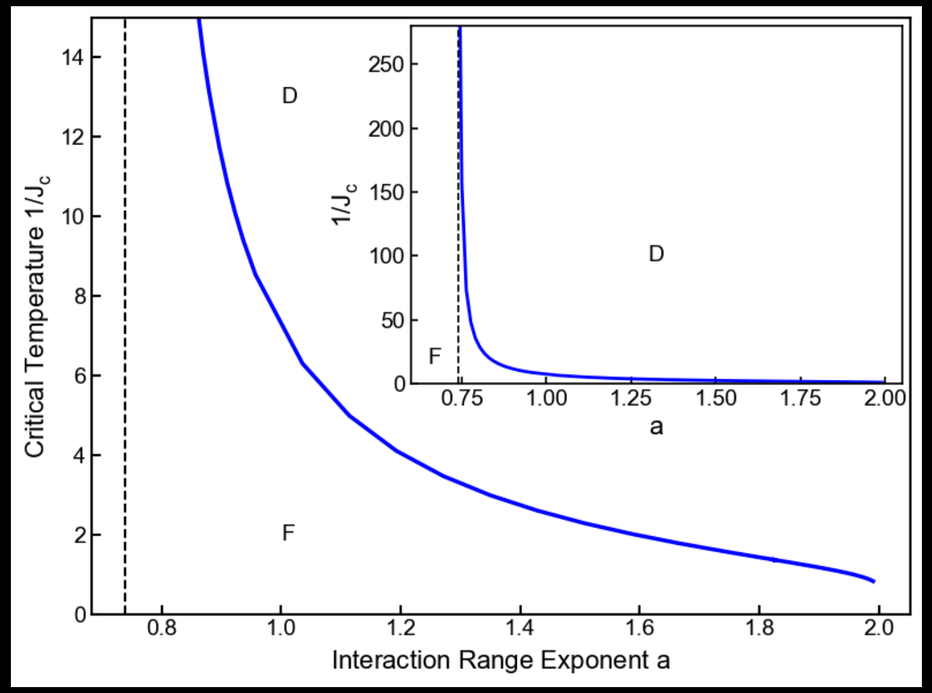
ISING MODEL

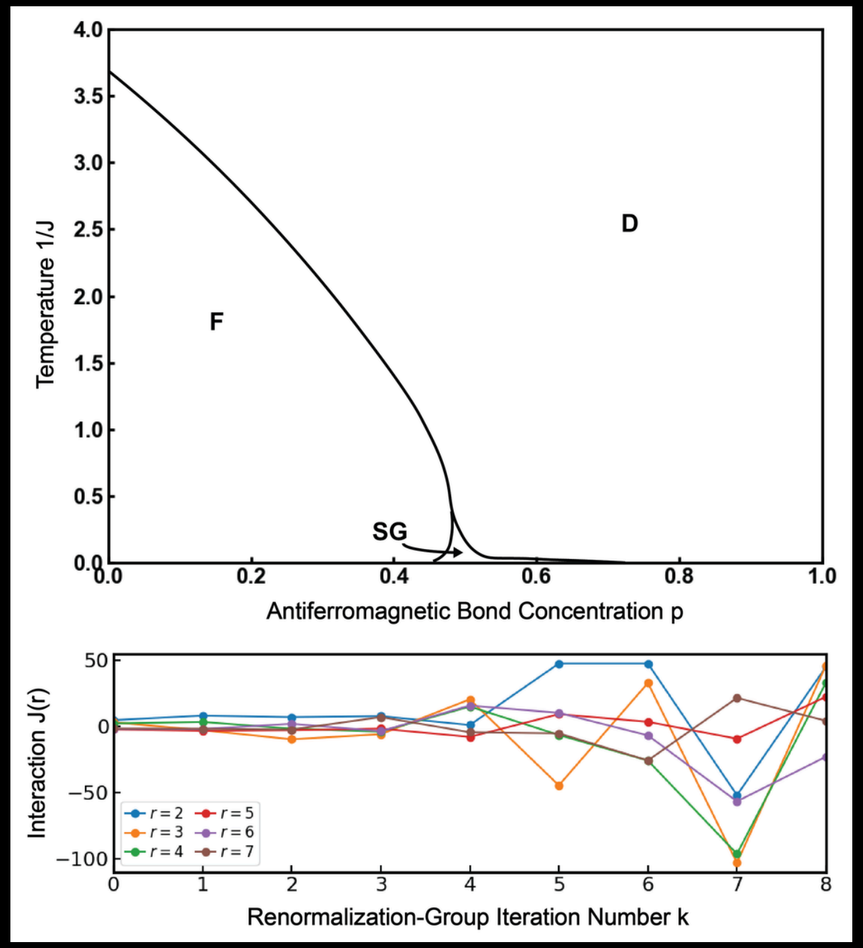




Long-Range Interactions

ISING MODEL



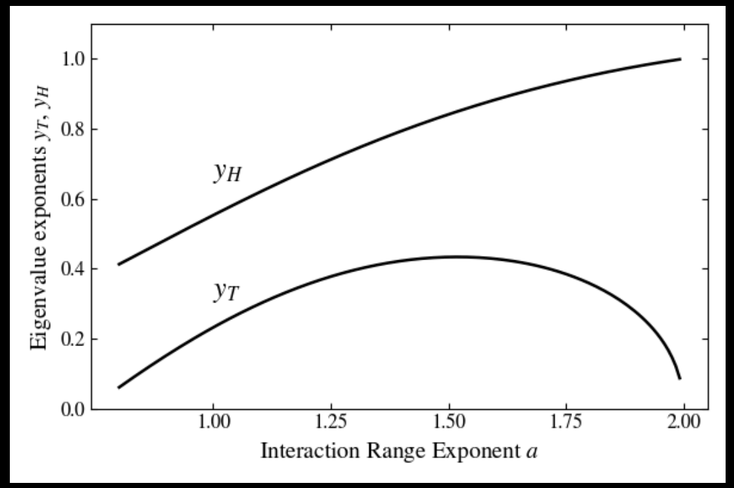


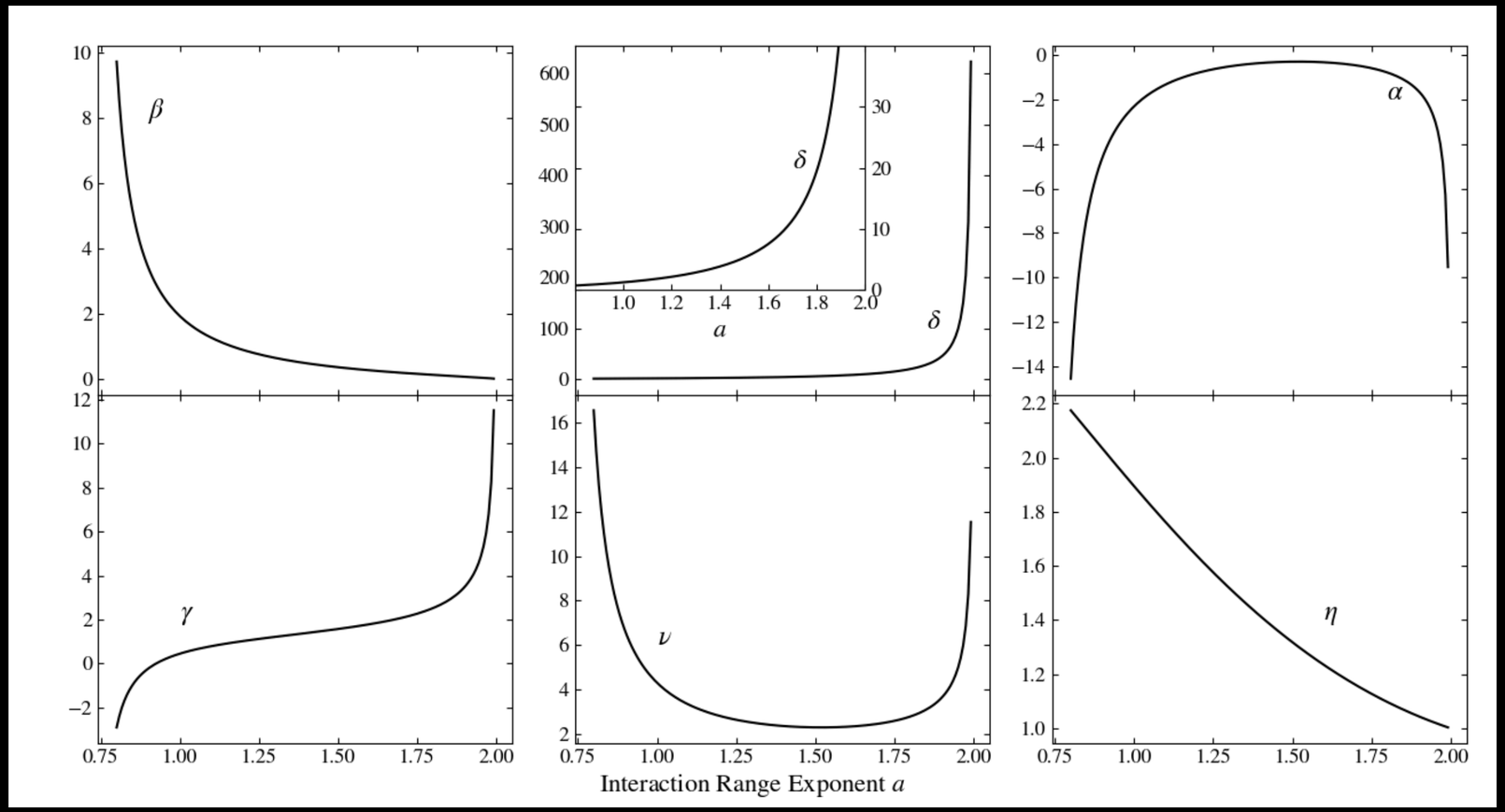
CRITICAL EXPONENTS AS FUNCTIONS OF RANGE

$$T = \partial J'_k / \partial J_l \longrightarrow \lambda_T = b^{y_T}$$

$$\partial H' / \partial H = b^{y_H}$$

$$\nu = 1/y_T$$
$$\alpha = 2 - d/y_T$$
$$\beta = (2 - y_H)/y_T$$
$$\delta = y_H/(d - y_H)$$
$$\gamma = (2y_H - d)/y_T$$
$$\eta = 2 + d - y_H$$





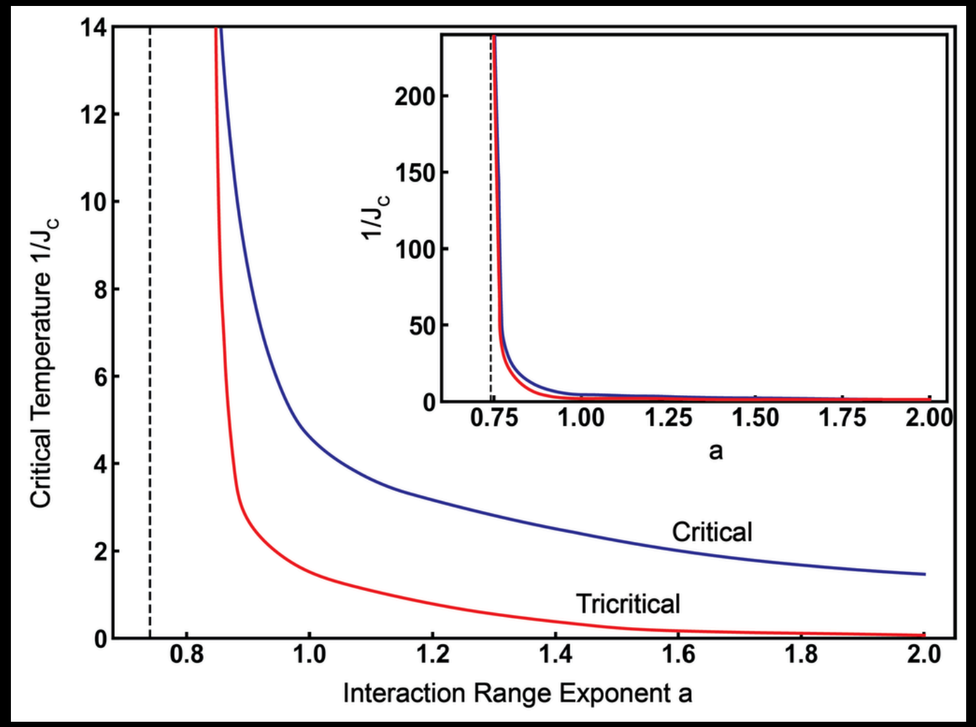
Long-Range Interactions

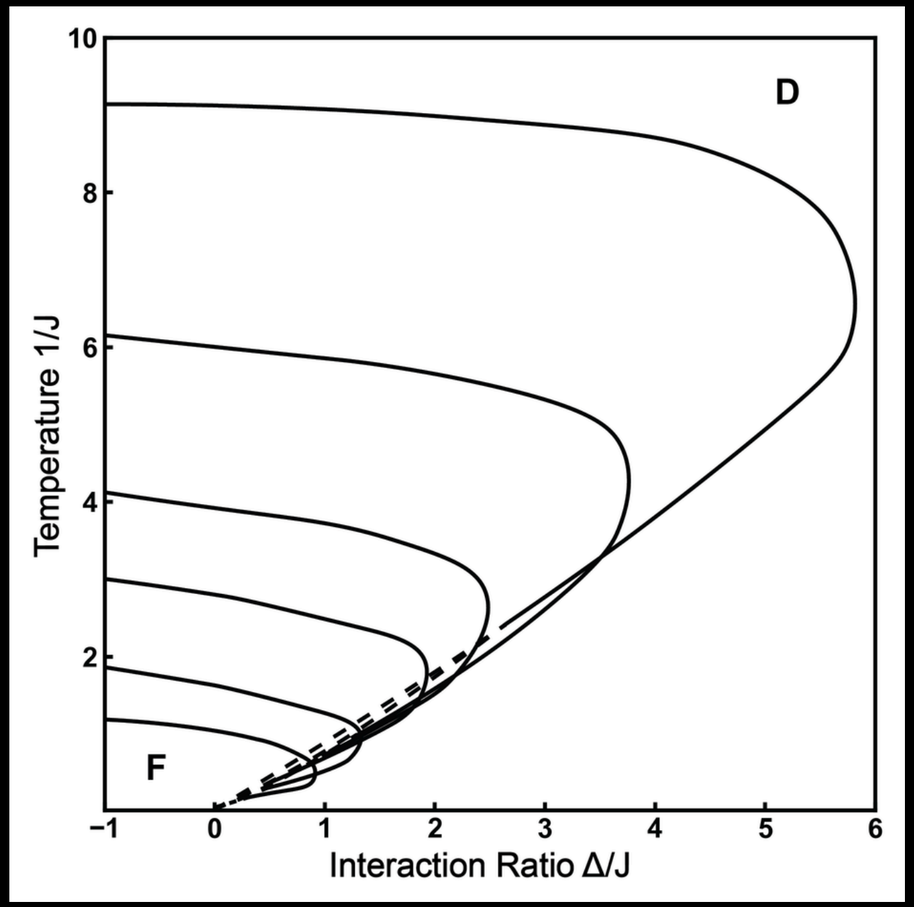
BLUME-CAPEL MODEL

$$-\beta\mathcal{H} = \sum_{r_1 \neq r_2} [J s_{r_1} s_{r_2} - \Delta(s_{r_1}^2 + s_{r_2}^2)] |r_1 - r_2|^{-a}$$

$$s'_{r'_i} = \pm 1, 0$$

BLUME-CAPEL MODEL





Metastable Phases

METASTABLE POTTS DROPLETS

$$-\beta\mathcal{H} = \sum_{<ij>}\Big\{\left[J\delta(s_i,s_j) - 1/q\right] + H\left[\delta(s_i,1) + \delta(s_j,1)\right]\Big\}$$

under RG:

$$-\beta\mathcal{H} = \sum_{\langle ij\rangle}\left[E(s_i,s_j) + G\right]$$

TRANSFER MATRIX IMPLEMENTATION

$$T(s_i,s_j) = e^{E(s_i,s_j)}$$

$$\tilde{T}(s_i,s_j) = \left[T(s_i,s_j)\right]^{b^{d-1}} \quad \text{(bond-moving)}$$

$$\mathbf{T}' = \tilde{\mathbf{T}}\cdot\tilde{\mathbf{T}}\,e^{\tilde{G}} \quad \text{(decimation)}$$

$$G' = b^d G + \tilde{G}$$

Metastable Phases

DENSITY RECURSION WITH FIXED BOUNDARY CONDITIONS

$$\mathbf{M} = b^{-d}\,\mathbf{M}' \cdot \mathbf{R} \longrightarrow \mathbf{M}^{(0)} = b^{-nd}\,\mathbf{M}^{(n)} \cdot \mathbf{R}^{(n)} \cdots \mathbf{R}^{(1)}$$

$$\mathbf{M} = [1, \langle \delta(s_i, m)\delta(s_j, n)\rangle]$$

$$\mathbf{K} = [G, E(m, n)]$$

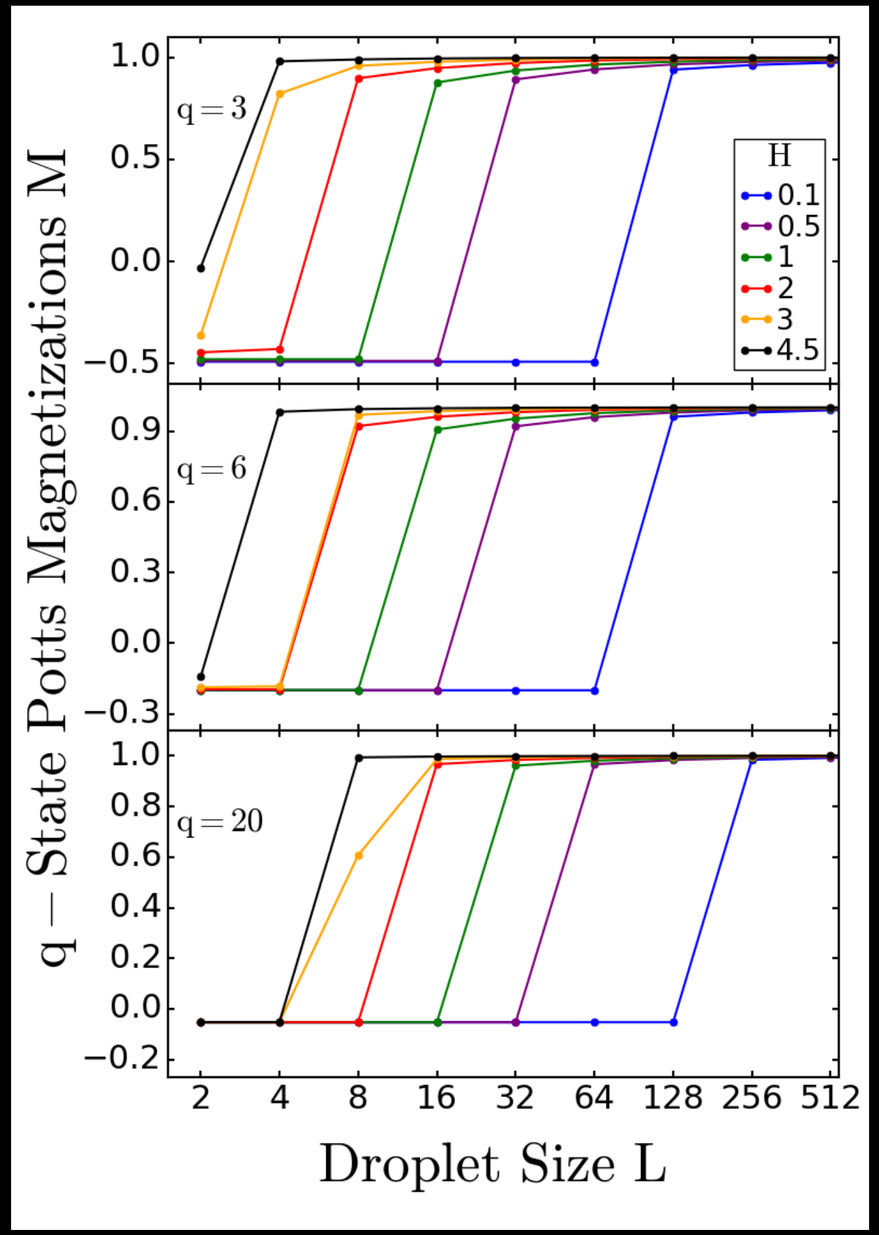
$$\mathbf{R} = \partial \mathbf{K}' / \partial \mathbf{K}$$

$$L = b^n$$

(droplet linear size)

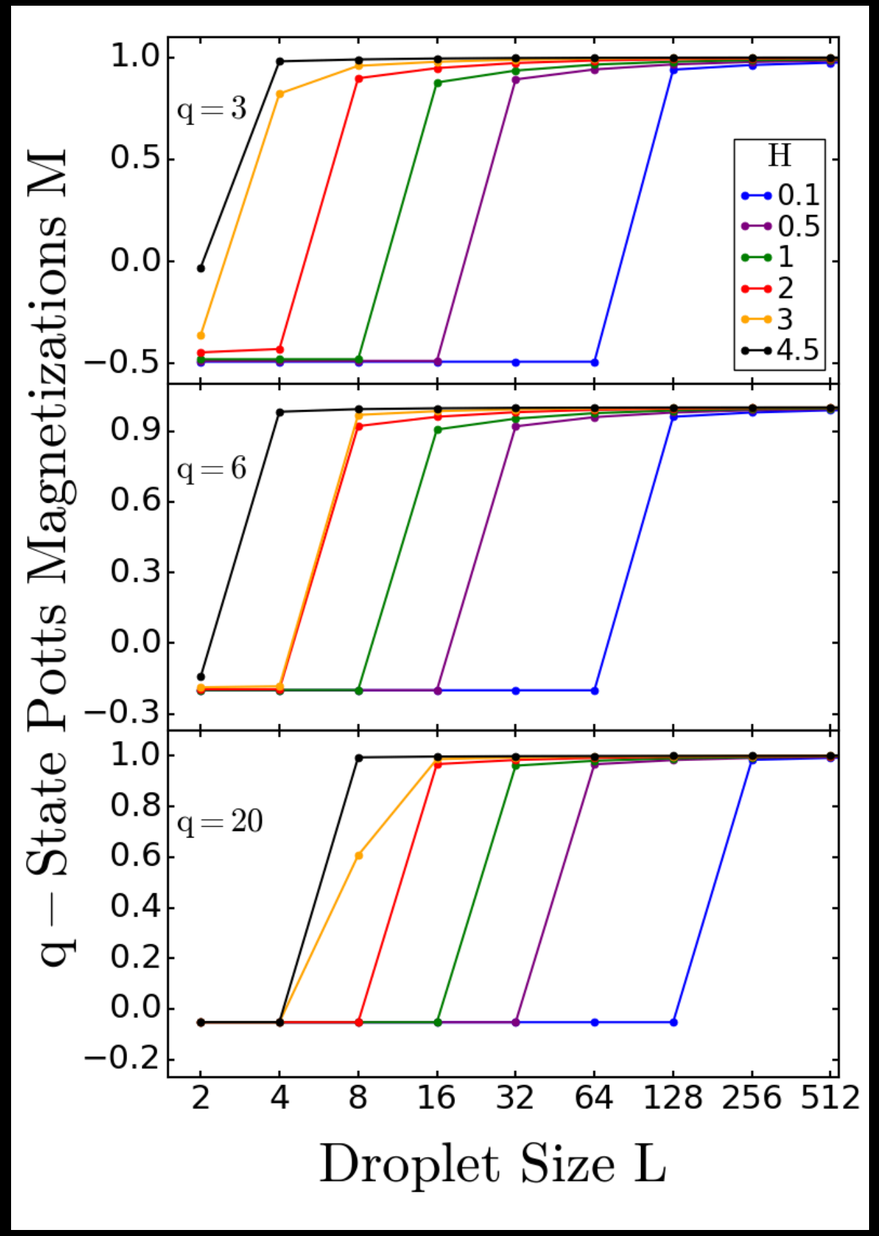
PROCEDURE

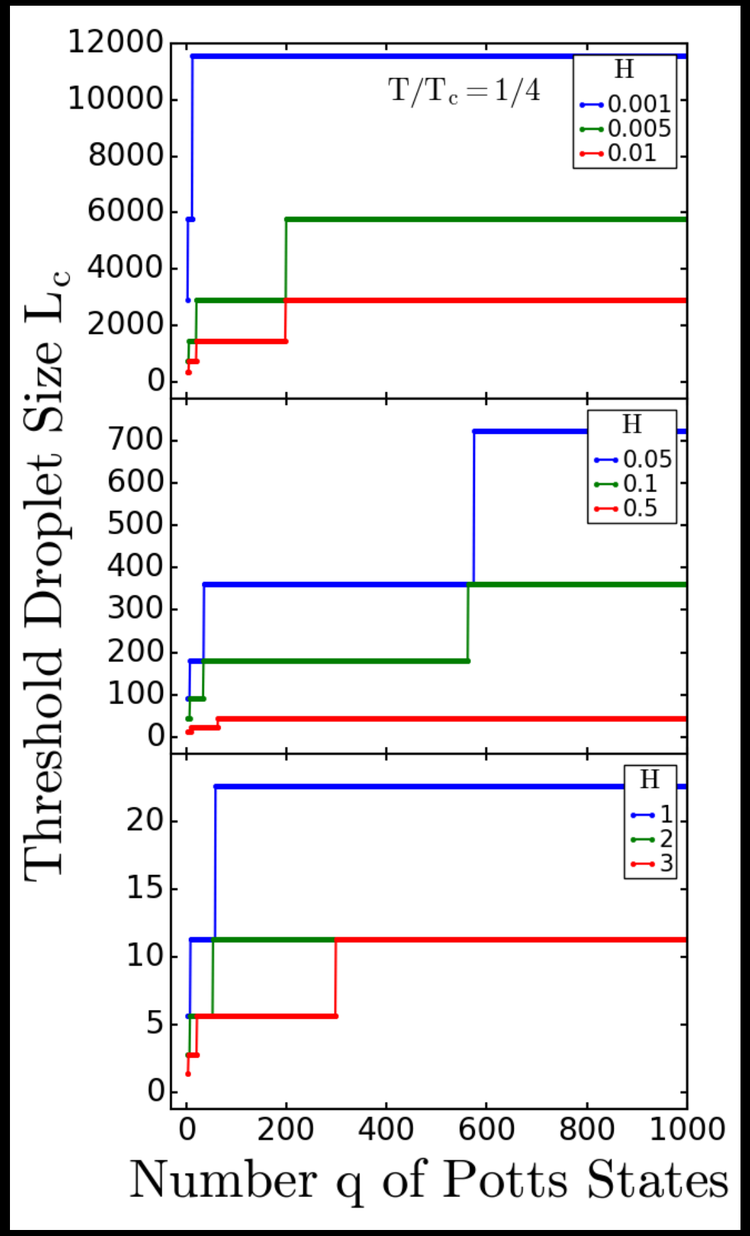
1. start with a droplet size $L = b^n$ with boundary spins fixed in the metastable phase (state $m \neq 1$)
2. apply n RG iterations, tracking how the interior magnetization evolves
3. if $M < 0$ at the end, the metastable droplet sustains; if $M > 0$, it does not
4. the critical size L_c is the largest L for which $M < 0$

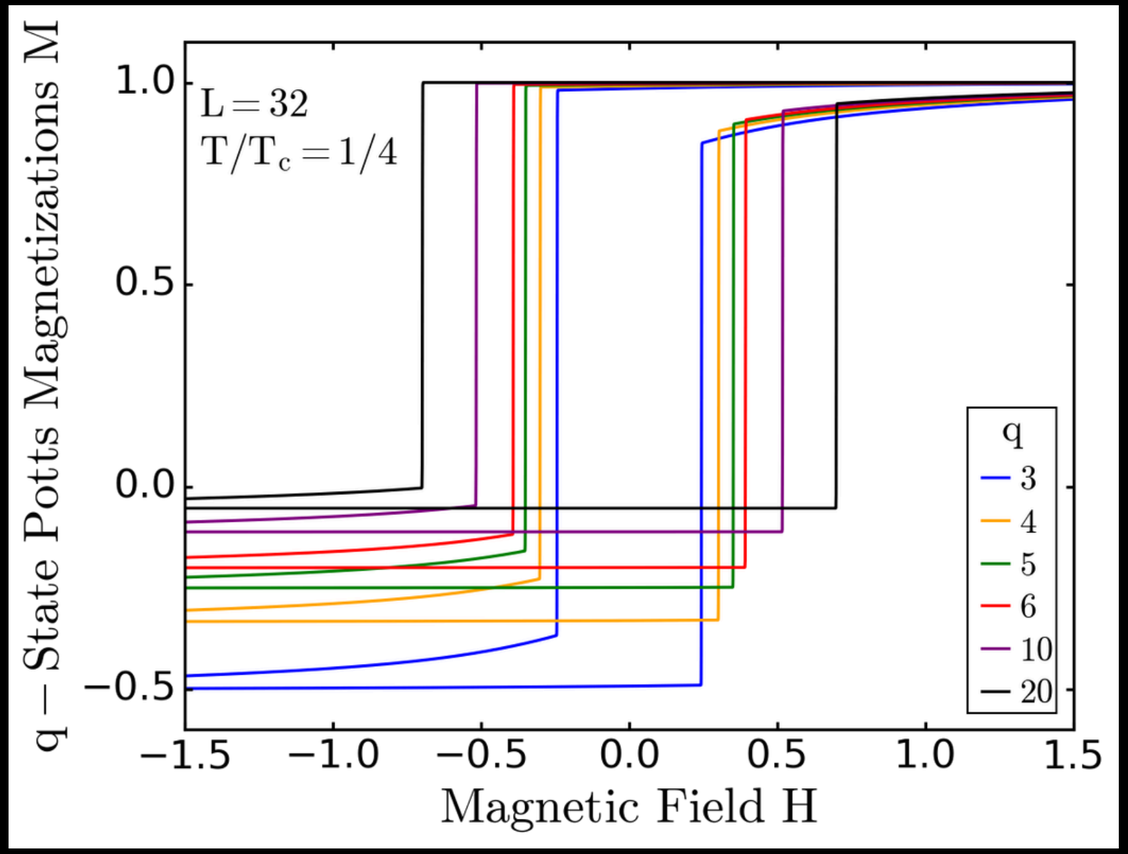


PROCEDURE

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q = 3
q = 6
q = 20
H
0.1
0.5
1
2
3
4.5
q − State Potts Magnetizations M
Droplet Size L

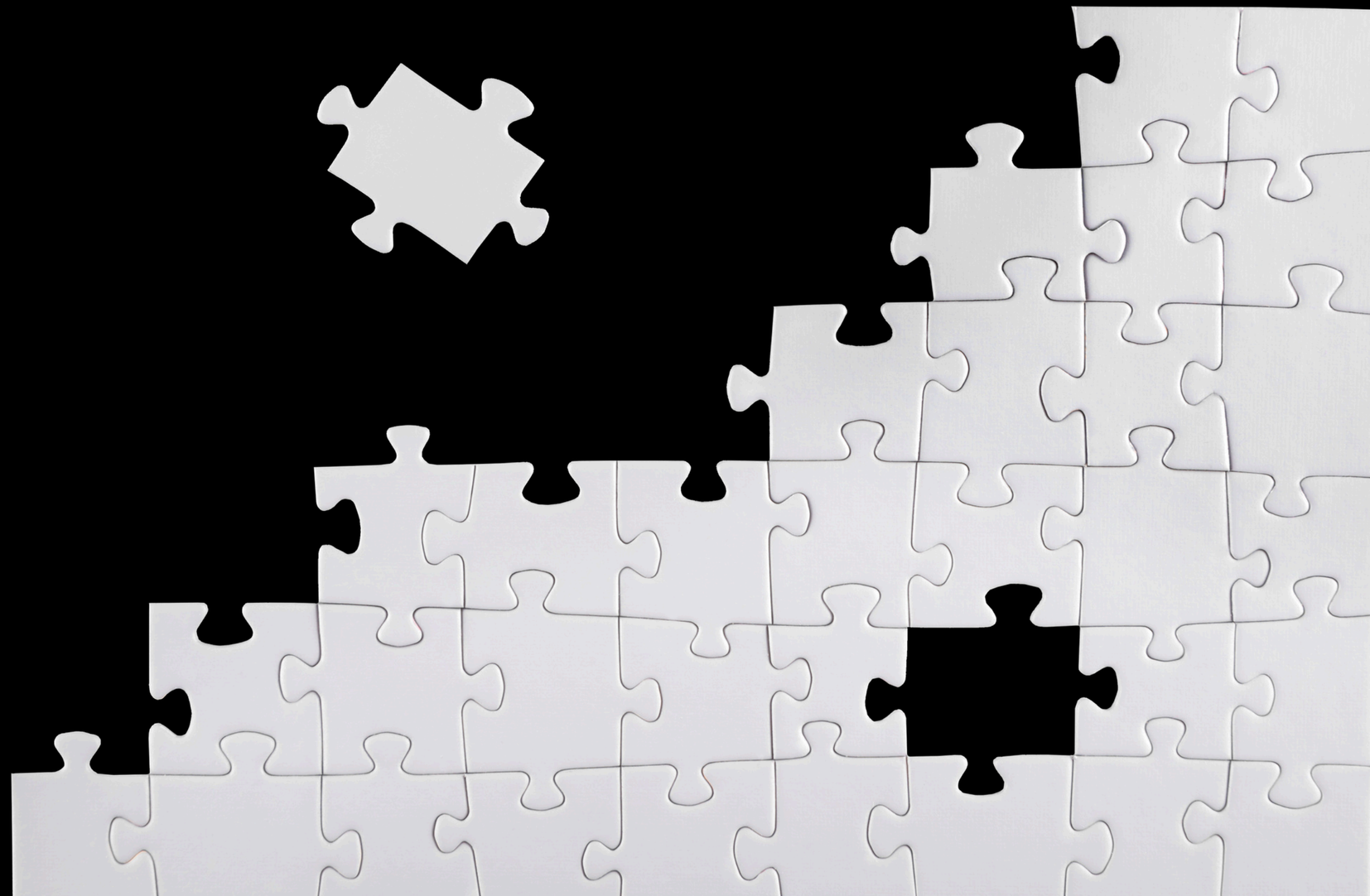

T/T_c = 1/4
H
0.001
0.005
0.01
0.05
0.1
0.5
1
2
3
Threshold Droplet Size L_c
Number q of Potts States


q − State Potts Magnetizations M
1.0
0.5
0.0
−0.5
L = 32
T/T_c = 1/4
q
3
4
5
6
10
20
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
Magnetic Field H

POTENTIAL APPLICATIONS

- surface-adsorbed systems, where metastable structures persist for experimental timescales
- water, where supercooled metastable regions may host criticality
- magnetic materials, where reversed domains and hysteresis are directly described by droplet stability

Synthesis & Perspective



Synthesis: RG Flow Topology as Organizing Principle

- fixed-point sinks --> conventional phases (ordered or disordered), with unstable fixed points controlling criticality via eigenvalues and eigenvectors
- fixed-line sinks --> algebraically ordered phases with continuously varying exponents across the phase
- strange attractors --> spin-glass phases, with Lyapunov and runaway exponents quantifying internal chaotic structure
- multiple attractors (of any type) --> multiple thermodynamic phases, with basin boundaries forming phase transitions

Perspective: Future of RG

RG studies are moving away from traditional equilibrium lattice models toward dynamical, out-of-equilibrium, and data-driven frameworks

- ML & computation: RG as an architectural principle; Delta-ML and DMRG, distributed computing, RG of eigen microstates
- quantum: from bulk properties to interfaces and monitored dynamics; sFRG, monitored fermion dynamics
- non-equilibrium & dynamical: the route to thermalization; NTFP, spontaneous stochasticity, turbulance
- non-spin: biological & social networks, dynamic percolation

thank you

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Questions & Discussion